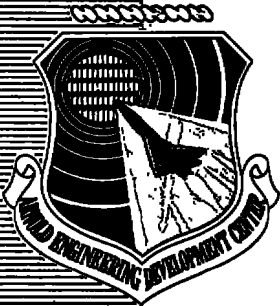


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VISCOUS SHOCK LAYER SOLUTIONS FOR HYPERSONIC SPHERE-CONES

DEPARTMENT OF AEROSPACE ENGINEERING
UNIVERSITY OF CINCINNATI
CINCINNATI, OHIO 45221

January 1977

Final Report for Period March 1976 — October 1976

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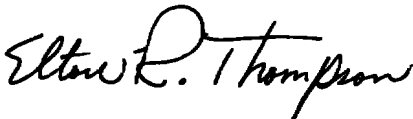
This final report was submitted by University of Cincinnati, Cincinnati, OH 45221 under contract F40600-74-C-0011, job order P00001, with the Arnold Engineering Development Center, Arnold Air Force Station, TN 37389. Mr. Elton R. Thompson, DYR, was the Air Force Project Engineer.

This report has been reviewed by the Information Office (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

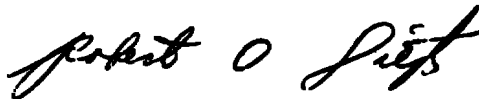
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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AEDC-TR-77-20	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) VISCOUS SHOCK LAYER SOLUTIONS FOR HYPERSONIC SPHERE-CONES		5. TYPE OF REPORT & PERIOD COVERED Final Report for Period - March - October 1976
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) B. N. Srivastava, M. J. Werle, and R. T. Davis		8. CONTRACT OR GRANT NUMBER(s) F40600-74-C-0011
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Aerospace Engineering University of Cincinnati Cincinnati, Ohio 45221		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Program Element 65807F
11. CONTROLLING OFFICE NAME AND ADDRESS Arnold Engineering Development Center (DYFS) Air Force Systems Command Arnold Air Force Station, Tennessee 37389		12. REPORT DATE January 1977
		13. NUMBER OF PAGES 180
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE N/A
16. DISTRIBUTION STATEMENT (of this Report) Approved for public release; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) <i>1. Sphere - Cones. 2. Blunt cones</i>		
18. SUPPLEMENTARY NOTES <i>3. Cones - Hypersonic flow.</i> Available in DDC		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) <div style="display: flex; justify-content: space-between;"> <div> blunt bodies flow fields hypersonic flow equations </div> <div> layers shock(mechanics) numerical analysis </div> </div>		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) An analysis and numerical solution of the fully viscous shock layer equations for hyper-sonic flow past spherically blunted cones is presented. Attention is drawn to the adverse effect in the numerical solution due to discontinuities in the flow derivatives caused by the discontinuity in the surface curvature at the sphere/cone tangency point, when the usual surface coordinate frame of reference is used. It is shown that a finite difference formulation that accounts for the imbedded gradient discontinuities resolves these nu-		

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20. ABSTRACT (Continued)

merical difficulties. This concept is demonstrated through a numerical scheme which utilizes a time dependent relaxation technique for the bow shock shape. A model problem analogous to the sphere/cone juncture problem is first formulated and finite difference schemes developed and demonstrated for this case. This approach is then extended to the solution of the viscous shock layer equations for hypersonic flow past spherically blunted cones with half cone angles varying from 30° to 0° at high Reynolds number. For these cases the present results are found to compare well with independent inviscid flow calculations. In addition, excellent comparisons with experimental pressure and heat transfer data on a 7.5° half angle cone are obtained.

PREFACE

The results reported herein were obtained from the Arnold Engineering Development Center by the University of Cincinnati, Department of Aerospace Engineering under Contract F40600-74-C-0011. The authors of this report were B. N. Srivastava, M. J. Werle, and R. T. Davis. The Air Force Project Engineer for this contract was E. R. Thompson, AEDC/DYR. The Program Element Number was 65807F.

This report covers the work during the period March 1976 to October 1976. The reproducibles used in the reproduction of this report were supplied by the authors.

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1. INTRODUCTION

Recent developments in aerodynamics and space flight have caused increasingly focused attention on the problem of theoretically predicting the blunt body flow field. Three current numerical approaches for treating this problem include solution of either the full Navier-Stokes equations, the second-order boundary-layer equations, or the viscous shock layer equations. Use of the full Navier-Stokes equations [1] has been quite successful in providing solutions for stagnation regions, but generally have been applied for only one nose radius downstream. This is because the elliptic nature of these equations increases the complexity of the solution procedure and restricts the application of these methods in the downstream direction. While there are several computational difficulties associated with the second-order boundary layer approach [2], many of the difficulties associated with computing viscous hypersonic flows over blunt bodies can be overcome through use of the viscous shock layer equations. In this approach the entire flow field, from the body to the shock, is treated in a unified manner. This is the approach taken here, wherein the basic method of Davis [2] is applied to nonanalytic bodies such as spherically blunted cones.

Blunted cones merit further study, both because some blunting of the tip of a sharp cone due to extreme heating appears unavoidable and because blunting has favorable effects on transition. Previous research [3, 4] has shown that a small amount of bluntness to a sharp cone is conducive to a delay in boundary layer transition. Unfortunately only limited work has been done in the past for such nonanalytic bodies, in part, due to the difficulties associated with the discontinuity occurring in the surface curvature at the sphere/cone juncture point whenever a surface coordinate system is used. Miner and Lewis [5] and Kang and Dunn [6] have been able to obtain some numerical solutions for such nose cone problems. The approach of Miner and Lewis [5] was to smoothen the effect of the curvature discontinuity at the sphere/cone tangency point by constructing an artificially continuous distribution of curvature. Kang and Dunn [6] approached the same problem by application of a Karman-Pohlhausen integral method to the Navier-Stokes equations under a thin shock layer assumption. They treated the nose region as well as the afterbody conical section of the spherically blunted cones in a similar manner. While these solutions yield acceptable numerical results, it appears that a more consistent formulation, which accounted for the flow behavior at the sphere cone tangency point, could improve these solutions and enhance their reliability.

The basic difficulty, however, is that only a very limited amount of information is available for the flow behavior at such a curvature discontinuity - information that could be utilized in a computational procedure to achieve more accurate solutions. It is known from inviscid considerations that a streamline curvature discontinuity produces a discontinuity in the flow gradients along that streamline. Viscous effects in such problems are expected to remove such surface gradient jumps in a manner shown by Messiter and Hu [7] for two dimensional flows. Their analysis for high Reynolds number flows shows that the jump in pressure gradient predicted by the inviscid flow theory is smoothed by viscous effects through a triple deck structure near the juncture region. However the region within which this smoothing takes place is found to be small for realistic finite Reynolds numbers and it would be anticipated that a viscous numerical calculation with a finite mesh size might not capture this physical phenomenon and would therefore still predict a gradient discontinuity. Thus, one is faced with the prospect of generating finite difference approximations for regions in which the flow gradients are virtually discontinuous.

An added complication arises because the viscous shock layer equations, when written in a surface coordinate

system, contain explicit dependence on the surface curvature. The surface curvature undergoes a discontinuous change from a value of one on the sphere to a value of zero on the cone portion resulting in the appearance of gradient discontinuities in all flow variables at all points normal to the body surface across the shock layer thickness at the juncture point. These discontinuities are purely an artifice and occur only because of the choice of surface coordinates. None-the-less they must be accounted for in any numerical approximating procedure if reliable results are to be achieved.

An analytical assessment of the flow behavior at the sphere/cone juncture point and subsequently a proper numerical formulation of this problem is the purpose of the present study. It is believed that when a surface coordinate frame of reference is used a finite difference formulation of the governing equations must be such that the longitudinal derivatives be carefully evaluated at the sphere/cone juncture point in order to eliminate large numerical truncation errors. This can be done by ensuring that the finite difference form of the longitudinal derivatives avoid any differencing across the sphere/cone juncture discontinuity. This technique is demonstrated in the present approach where the numerical scheme utilizes a

time dependent relaxation technique for the shock wave shape. A model problem analogous to the sphere/cone juncture problem is first formulated and finite difference schemes developed and demonstrated for this case. It is shown that the present method accurately captures the anticipated discontinuous behavior of the flow derivatives at the model juncture point. This concept is then extended for the solution of the viscous shock layer equations for hypersonic flow past spherically blunted cones with half cone angles varying from 30° to 0° under various free stream conditions. The results indicate good comparisons with inviscid solutions and experimental data.

II. GOVERNING EQUATIONS

The viscous shock layer concept has been presented in detail by Davis [2] and therefore is only summarized here. The compressible Navier-Stokes equations are written in a boundary layer like coordinate system (see Fig. 1) and nondimensionalized by variables which are of order one in the region near the body surface (boundary layer) for large Reynolds numbers. The same set of equations are then written in variables which are of order one in the essentially inviscid region outside the boundary layer. In the final set of equations terms are retained that are second-order in the inverse square root of a Reynolds number. A comparison of the two sets of equations is then made and one set of equations is found from them which is valid to second order in both the (inviscid) outer and inner (viscous) regions. A solution to this set of equations is thus uniformly valid to second order in the entire shock layer for arbitrary Reynolds number. The resulting equations (and notation) are the same as those presented by Davis [2] and are given as:

Continuity:

$$[(r+n\cos\phi)^j \rho u]_s + [(1+\kappa n)(r+n\cos\phi)^j \rho v]_n = 0 \quad (1a)$$

Longitudinal Momentum:

$$\begin{aligned} \rho \{ u u_s / (1 + \kappa n) + v u_n + \kappa u v / (1 + \kappa n) \} + p_s / (1 + \kappa n) \\ = [\epsilon^2 / (1 + \kappa n)^2 (r + n \cos \phi)^j] [(1 + \kappa n)^2 (r + n \cos \phi)^j \tau]_n \end{aligned} \quad (1b)$$

where,

$$\tau = \mu [u_n - \kappa u / (1 + \kappa n)] \quad (1c)$$

Normal Momentum:

$$\rho \{ u v_s / (1 + \kappa n) + v v_n - \kappa u^2 / (1 + \kappa n) \} + p_n = 0 \quad (1d)$$

which with the thin shock layer approximation becomes,

$$-\rho \kappa u^2 / (1 + \kappa n) + p_n = 0 \quad (1e)$$

Energy Equation:

$$\begin{aligned} \rho \{ u T_s / (1 + \kappa n) + v T_n \} - u p_s / (1 + \kappa n) - v p_n = \epsilon^2 \tau^2 / \mu \\ + [\epsilon^2 / (1 + \kappa n) (r + n \cos \phi)^j] [(1 + \kappa n) (r + n \cos \phi)^j q]_n \end{aligned} \quad (1f)$$

where

$$q = \mu T_n / \sigma \quad (1g)$$

Equation of State:

$$p = (\gamma - 1) \rho T / \gamma \quad (1h)$$

Viscosity Law:

$$\mu = T^{3/2} (1 + c') / (T + c') \quad (1i)$$

where

$$c' = c^*/M_\infty^2 T_\infty^* (\gamma-1)$$

and c^* is taken to be 198.6°R for air.

The boundary conditions employed here are the no slip surface conditions,

$$u(s,0) = v(s,0) = 0 \quad (2a)$$

and

$$T(s,0) = T_w \quad (2b)$$

while at the shock location the oblique shock relations are used to relate the flow variables just aft of the shock to the free stream conditions through the local shock slope. These relations are given as

$$u_{sh} = \tilde{u}_{sh} \sin(\alpha+\beta) + \tilde{v}_{sh} \cos(\alpha+\beta) \quad (3a)$$

$$v_{sh} = -\tilde{u}_{sh} \cos(\alpha+\beta) + \tilde{v}_{sh} \sin(\alpha+\beta) \quad (3b)$$

where \tilde{u}_{sh} and \tilde{v}_{sh} are velocity components at the shock given as

$$\tilde{u}_{sh} = \cos\alpha \quad (3c)$$

$$\tilde{v}_{sh} = -\sin\alpha/\rho_{sh} \quad (3d)$$

and

$$\rho_{sh} = \gamma p_{sh}/(\gamma-1)T_{sh} \quad (3e)$$

with

$$p_{sh} = [2/(\gamma+1)]\sin^2\alpha - (\gamma-1)/\gamma(\gamma+1)M_\infty^2 \quad (3f)$$

$$T_{sh} = (\tilde{u}_{sh} - \cos\alpha)^2/2 + \{[4\gamma/(\gamma+1)^2]\sin^2\alpha + [2/(\gamma-1) - 4(\gamma-1)/(\gamma+1)^2]/M_\infty^2 - 4/(\gamma+1)^2 M_\infty^4 \sin^2\alpha\}/2 \quad (3g)$$

The shock angle, α , is related to the shock thickness, n_s , through the relation

$$\frac{dn_s}{ds} / (1 + \kappa n_s) = \tan(\alpha - \phi) \quad (4a)$$

The value of n_s itself can be written from mass conservation considerations as

$$(r + n_s \cos\phi)^{1+j} = 2^j \int_0^{n_s} \rho u (r + n \cos\phi)^j dn \quad (4b)$$

For reasons explained in Reference [2], the above equations will be normalized according to the following scheme:

$$\begin{aligned} \eta &= n/n_{sh} & \xi &= s & \bar{u} &= u/u_{sh} & \bar{v} &= v/v_{sh} \\ \bar{t} &= T/T_{sh} & \bar{p} &= p/p_{sh} & \bar{\rho} &= \rho/\rho_{sh} & \bar{\mu} &= \mu/\mu_{sh} \end{aligned} \quad (5a-h)$$

The differential relations needed to transform equations (1a) through (1i) are given by

$$\partial/\partial s = \partial/\partial \xi - \eta (n'_{sh}/n_{sh}) \partial/\partial \eta$$

$$\partial/\partial n = (1/n_{sh}) \partial/\partial \eta$$

$$\partial^2/\partial n^2 = (1/n_{sh}^2) \partial^2/\partial \eta^2 \quad (6)$$

where,

$$n'_{sh} = (dn_{sh}/d\xi)$$

The s-momentum and energy equations (1b) and (1f) written in the transformed ξ, η plane can be conveniently put in a standardized form for a parabolic equation as,

$$\partial^2 w / \partial \eta^2 + \alpha_1 (\partial w / \partial \eta) + \alpha_2 w + \alpha_3 + \alpha_4 (\partial w / \partial \xi) = 0 \quad (7)$$

where w represents \bar{u} for the s-momentum equation, and \bar{t} for the energy equation. For the momentum equation the coefficients $\alpha_1 \rightarrow \alpha_4$ are:

$$\begin{aligned} \alpha_1 = & \frac{\rho_{sh} u_{sh} n'_{sh}}{\epsilon^2 \mu_{sh}} \frac{n_{sh}}{1 + \kappa n_{sh} \eta} \frac{\bar{\rho} \bar{u} \eta}{\bar{\mu}} - \frac{\rho_{sh} v_{sh} n_{sh}}{\epsilon^2 \mu_{sh}} \frac{\bar{\rho} \bar{v}}{\bar{\mu}} \\ & + \frac{\bar{\mu}_\eta}{\bar{\mu}} + \frac{\kappa n_{sh}}{1 + \kappa n_{sh} \eta} + \frac{\cos \phi n_{sh}}{\gamma + n_{sh} \eta \cos \phi} \end{aligned} \quad (8a)$$

$$\begin{aligned}
\alpha_2 = & - \frac{\rho_{sh} u_{sh} n_{sh}}{\epsilon^2 \mu_{sh}} \frac{n_{sh}}{1+\kappa n_{sh} \eta} \frac{\bar{\rho} \bar{u}}{\bar{\mu}} - \frac{\rho_{sh} v_{sh} n_{sh}}{\epsilon^2 \mu_{sh}} \frac{\kappa n_{sh}}{1+\kappa n_{sh} \eta} \frac{\bar{\rho}}{\bar{\mu}} \frac{\bar{v}}{\bar{\mu}} \\
& - \kappa \frac{n_{sh}}{1+\kappa n_{sh} \eta} \frac{\bar{\mu}}{\bar{\mu}} \frac{1}{\bar{\mu}} - \left(\frac{\kappa n_{sh}}{1+\kappa n_{sh} \eta} + \frac{\cos \phi n_{sh}}{r+n_{sh} \eta \cos \phi} \right) \times \left(\frac{\kappa n_{sh}}{1+\kappa n_{sh} \eta} \right)
\end{aligned} \quad (8b)$$

$$\alpha_3 = - \frac{p_{sh} n_{sh}}{\epsilon^2 \mu_{sh}} \frac{n_{sh}}{1+\kappa n_{sh}} \frac{1}{\bar{\mu}} \frac{1}{u_{sh}} (\bar{p}_\xi - \frac{n_{sh}}{n_{sh}} \eta \bar{p}_\eta + \frac{p_{sh}}{p_{sh}} \bar{p}) \quad (8c)$$

$$\alpha_4 = - (\rho_{sh} u_{sh} n_{sh} / \epsilon^2 \mu_{sh}) (n_{sh} / (1+\kappa n_{sh} \eta)) \frac{\bar{\rho} \bar{u}}{\bar{\mu}} \quad (8d)$$

For the energy equation the coefficients are

$$\begin{aligned}
\alpha_1 = & \frac{\rho_{sh} u_{sh} n_{sh}^\sigma}{\epsilon^2 \mu_{sh}} \frac{n_{sh}}{1+\kappa n_{sh} \eta} \frac{\bar{\rho} \bar{u} \eta}{\bar{\mu}} - \frac{\rho_{sh} v_{sh} n_{sh}^\sigma}{\epsilon^2 \mu_{sh}} \times \frac{\bar{\rho} \bar{v}}{\bar{\mu}} \\
& + \frac{\bar{\mu}}{\bar{\mu}} \frac{1}{\bar{\mu}} + \frac{\kappa n_{sh}}{1+\kappa n_{sh} \eta} + \frac{\cos \phi n_{sh}}{r+n_{sh} \eta \cos \phi}
\end{aligned} \quad (9a)$$

$$\alpha_2 = - (\rho_{sh} u_{sh} T_{sh} / \epsilon^2 \mu_{sh} T_{sh}) (n_{sh}^2 / (1+\kappa n_{sh} \eta)) \times \frac{\bar{\rho} \bar{u}}{\bar{\mu}} \quad (9b)$$

$$\begin{aligned}
\alpha_3 = & \frac{\rho_{sh} u_{sh} n_{sh}^\sigma}{\epsilon^2 \mu_{sh} T_{sh}} \frac{1}{\bar{\mu}} \left[\frac{n_{sh} \bar{u}}{1+\kappa n_{sh} \eta} (\bar{p}_\xi - \frac{n_{sh}}{n_{sh}} \eta \bar{p}_\eta + \frac{p_{sh}}{p_{sh}} \bar{p}) \right. \\
& \left. + \frac{v_{sh}}{u_{sh}} \bar{v} \bar{p}_\eta \right] + \frac{u_{sh}^2}{T_{sh}} (\bar{u}_\eta - \frac{\kappa n_{sh}}{1+\kappa n_{sh} \eta} \bar{u})^2
\end{aligned} \quad (9c)$$

$$\alpha_4 = -(\sigma \rho_{sh} u_{sh} n_{sh} / \epsilon^2 \mu_{sh}) (n_{sh} / (1 + \kappa n_{sh} \eta)) \frac{\bar{\rho} \bar{u}}{\bar{\mu}} \quad (9d)$$

The remaining differential equations are first order and are the continuity equation:

$$[n_{sh} (r + n_{sh} \eta \cos \phi) \rho_{sh} u_{sh} \bar{\rho} \bar{u}]_{\xi} + [(r + n_{sh} \eta \cos \phi) \times \\ \{ (1 + \kappa n_{sh} \eta) \rho_{sh} v_{sh} \bar{\rho} \bar{v} - n'_{sh} \rho_{sh} u_{sh} \bar{\rho} \bar{u} \eta \}]_{\eta} = 0 \quad (10)$$

and the n-momentum equation:

$$\frac{\bar{\rho} \bar{u}}{(1 + \kappa n_{sh} \eta)} (\bar{v}_{\xi} - n'_{sh} / n_{sh} \eta \bar{v}_{\eta} + \frac{v'_{sh}}{v_{sh}} \bar{v}) + \frac{v_{sh}}{u_{sh}} \frac{\bar{\rho} \bar{v}}{n_{sh}} \bar{v}_{\eta} \\ - \frac{\kappa}{1 + \kappa n_{sh} \eta} \frac{u_{sh}}{v_{sh}} \bar{\rho} \bar{u}^2 + \frac{p_{sh}}{\rho_{sh} u_{sh} v_{sh} n_{sh}} \bar{p}_{\eta} = 0 \quad (11)$$

where with the thin layer approximation this equation becomes,

$$\bar{p}_{\eta} = [\kappa / (1 + \kappa n_{sh} \eta)] (\rho_{sh} u_{sh}^2 n_{sh} / p_{sh}) \bar{\rho} \bar{u}^2 \quad (12)$$

This leaves the equation of state,

$$\bar{p} = \bar{\rho} \bar{t} \quad (13)$$

and the viscosity law,

$$\bar{\mu} = [(T_{sh} + c') / (T_{sh} \bar{t} + c')] \bar{t}^{3/2} \quad (14)$$

At the shock location all variables are unity,

$$\bar{u} = \bar{v} = \bar{t} = \bar{p} = \bar{\rho} = \bar{\mu} = 1 \quad \text{at} \quad \eta = 1 \quad (15)$$

An equation of mass conservation can be obtained from equation (10) by integrating from $\eta = 0$ to $\eta = 1$ while holding ξ constant. This results in

$$\frac{dm}{d\xi} = (r+n_{sh}\cos\phi) [n_{sh}'\rho_{sh}u_{sh} - (1+\kappa n_{sh})\rho_{sh}v_{sh}] \quad (16a)$$

where

$$m = \int_0^1 n_{sh} (r+n_{sh}\eta\cos\phi) \rho_{sh} u_{sh} \bar{\rho} \bar{u} d\eta \quad (16b)$$

is proportional to the rate of mass flux between the body and shock at a given position on the body surface.

Equations (7), (10)-(14) and (16), constitute the complete set of governing equations for the unknowns \bar{u} , \bar{v} , \bar{t} , \bar{p} , $\bar{\mu}$, $\bar{\rho}$ and n_{sh} . These equations are solved along with the surface boundary conditions given by equations (2a) and (2b) and the shock conditions given by equation (15). The mass conservation equation (16a) and (16b) is used to determine the shock stand off distance n_{sh} . The general procedure is to evaluate the rate of mass flux between the body and shock at a given position on the body surface from equation (16b) using the values of the physical quantities previously calculated and then evaluate n_{sh} from equation (16a).

III. JUNCTURE REGION

1. Inviscid Flow Analysis

The aim of the present section is to analyze the nature of the flow at a sphere/cone juncture point in the light of various forms of the gas dynamic equations. It is of interest to analyze the viscous as well as inviscid gas dynamic equations in order to understand the physical behavior at a point in the flow field where a discontinuity in curvature is encountered. In order to be fully consistent in the present analysis, it is desirable to first consider the flow behavior from an inviscid standpoint and then subsequently include viscous effects.

From an inviscid standpoint, it is known from Euler's equations that a streamline curvature discontinuity produces a discontinuity in the flow gradients only along that streamline [8]. For locally supersonic flows, the discontinuity at a juncture point (e.g. sphere/cone tangency point) in the flow gradients will be propagated along the characteristic lines inclined at the local Mach angle to the flow direction. Eventually, therefore, such discontinuities in the flow gradients would propagate everywhere within the flow field downstream of the juncture discontinuity due to successive reflections of these characteristic lines from the shock and body surface.

Note, however, that at the juncture point the flow gradients will be continuous across the shock layer except at the body surface where gradient discontinuities will be present.. Note must also be made here that the flow variables themselves are found to be continuous at the juncture point across the shock layer. However, an added complication arises due to the explicit appearance of the surface curvature in these equations when written in a surface coordinate system. Since the surface curvature itself is discontinuous at the sphere/cone juncture point, it is necessary to assess the influence of the discontinuity on the flow properties and their derivatives. Intuitively it is obvious that a mere coordinate transformation would not affect the physical behavior so that the flow variables themselves are continuous at the juncture point all across the shock layer. This can also be straightforwardly demonstrated through consideration of the integral form of the conservation laws in the surface coordinate system as shown in Appendix (A).

Note that the set of equations (A7 , 14, 20) in Appendix (A) provide for either the trivial case of $p_1 = p_2$ and $u_1 = u_2$, or a shock like discontinuity. Since there is no physical event that could cause a shock at the sphere/cone juncture point, it is fairly evident that the trivial

solution is expected for this case indicating that the flow variables are continuous at the sphere/cone juncture point in the surface coordinate system. However the same conclusion does not apply to the flow derivatives with respect to the surface distance. This is evident from the differential form of the two-dimensional inviscid gas dynamic equations as recovered from the integral equations. These are given as,

Continuity

$$(\rho u)_s + (1+\kappa n)(\rho v)_n + \kappa \rho v = 0 \quad (17)$$

s-Momentum

$$p_s + \rho u u_s + (1+\kappa n) \rho v u_n + \kappa \rho u v = 0 \quad (18)$$

n-Momentum

$$\rho u v_s / (1+\kappa n) + \rho v v_n - \kappa \rho u^2 / (1+\kappa n) + p_n = 0 \quad (19)$$

Energy Equation

$$\rho u T_s + \rho v T_n (1+\kappa n) - u p_s - v p_n (1+\kappa n) = 0 \quad (20)$$

Equation of State

$$p = \left(\frac{\gamma-1}{\gamma} \right) \rho T \quad (21)$$

Using the equation of state and defining $a^2 = (\gamma p / \rho)$, the energy equation can be rewritten as,

$$u p_s + v (1+\kappa n) p_n - u a^2 \rho_s - v (1+\kappa n) a^2 \rho_n = 0 \quad (22)$$

Equations (17) through (22) can now be used on the two sides of the sphere/cone juncture point, noting that the surface curvature, κ , takes a value of 1.0 on the spherical part and a value of 0 on the conical part. Using subscript 1 and 2 for the sphere and cone portions respectively and noting that the flow variables are continuous across the juncture point, the inviscid equations provide the following jump conditions at the juncture point,

$$\rho_{s_1} = (1+n) \rho_{s_2} \quad (23a)$$

$$p_{s_1} = (1+n) p_{s_2} \quad (23b)$$

$$u_{s_1} = (1+n) u_{s_2} - v \quad (23c)$$

$$v_{s_1} = (1+n) v_{s_2} + u \quad (23d)$$

In addition to the flow variables themselves, consideration must also be given to the bow shock shape. Note that for problems of this type the shock shape itself would be expected to be smooth through the sphere cone juncture point with the shock slope at any location given in terms of the axial coordinate system (Figure 1) as

$$\tan \alpha = \frac{dR}{dx} \quad (24)$$

Since the shock shape, R , itself is a smooth function through the sphere/cone tangency point, the shock slope, α , would be smooth at this point. However, the first derivative of the shock distance, R , with respect to the surface coordinate system is obtained from the geometrical relation,

$$\frac{dR}{ds} = (1 + \kappa n_s) \frac{\sin \alpha}{\cos(\alpha - \phi)} \quad (25)$$

Since the shock angle, α , and the body angle, ϕ , are continuous functions of the surface distance, this relation yields a jump condition for dR/ds at the sphere/cone tangency point as,

$$\left(\frac{dR}{ds}\right)_{\text{sphere}} = (1 + n_s) \left(\frac{dR}{ds}\right)_{\text{cone}} \quad (26)$$

Similar discontinuous behavior can be shown to appear in derivatives such as dn_s/ds and dx_s/ds as shown by the following expressions,

$$\frac{dn_s}{ds} = (1 + \kappa n_s) \tan(\alpha - \phi) \quad (27)$$

$$\frac{dx_s}{ds} = (1 + \kappa n_s) \frac{\cos \alpha}{\cos(\alpha - \phi)} \quad (28)$$

The corresponding jump conditions, therefore, can be written as,

$$\left(\frac{dn_s}{ds}\right)_{\text{sphere}} = (1+n_s) \left(\frac{dn_s}{ds}\right)_{\text{cone}} \quad (29)$$

$$\left(\frac{dx_s}{ds}\right)_{\text{sphere}} = (1+n_s) \left(\frac{dx_s}{ds}\right)_{\text{cone}} \quad (30)$$

It is of interest to note that the expression (24), can be rewritten using the surface-coordinate system as,

$$\frac{dx_s}{ds} \tan \alpha = \frac{dR}{ds}$$

indicating that the discontinuity associated with dx_s/ds and dR/ds at the juncture point are of the nature such that the shock slope, α , itself is continuous at this point.

It must be pointed out here that similar jump conditions can also be established for the second derivatives of the flow quantities mentioned above, whenever required. These derivatives are undefined at the juncture point in this coordinate system, however finite values exist for these quantities immediately ahead and behind the juncture point. A typical case where higher derivatives are needed is at the shock location. The derivatives of the flow properties behind the shock are shown in Appendix (B). Note that these derivatives (B11, B15-17) undergo discontinuous changes at the sphere/cone juncture point and the magnitude of these discontinuities are related to the

surface curvature, κ , and the discontinuity associated with the second derivative of the shock shape, d^2R/ds^2 . However it is important to emphasize, here, that this situation is entirely due to the use of the surface coordinate system.

It is, thus, found that within the framework of Euler's equations the flow properties are continuous whereas the flow derivatives with respect to surface distance are discontinuous across the layer at the sphere/cone juncture point. The slope of the shock relative to the surface distance is also found to be discontinuous at this point.

2. Viscous Flow Analysis

(a) Classical Boundary Layer

From a viscous standpoint it seems more rational to first address the question of validity of the various forms of the viscous gas dynamic equations as applied to the spherically blunted cones with a discontinuous surface curvature at the juncture point. The boundary layer version of the Navier-Stokes equations cannot hold at the sphere/cone juncture point because the longitudinal derivative of the surface curvature, $\partial\kappa/\partial s \gg 1$ [9, 10] and thus the gradient of the corresponding inviscid pressure is discontinuous there. Any such discontinuity

would seem to be in violation of the boundary layer scaling laws wherein longitudinal derivatives are assumed to be much smaller than normal derivatives. This becomes more apparent when one considers inclusion of higher order terms in the boundary layer equations. The second order correction (in $Re^{-1/2}$) due to longitudinal curvature is driven by the rate of change of curvature - thus causing this higher order effect to rise up to first order level near a sphere/cone juncture point. It is, therefore, apparent that a new local solution needs to be developed near the juncture point in order to accommodate this anomaly. Such an analysis has been performed by Messiter and Hu [7] for two-dimensional flows.

(b) Triple Deck Analysis

A study of the juncture region has been completed by Messiter and Hu [7] for two-dimensional flow problems. They point out that, unlike the classical boundary layer case, an interaction with the external flow must be taken into account, this occurring through a small pressure change acting over a suitably small distance along the boundary layer. The details of the resulting local pressure distribution cannot be specified in advance, but must be found by studying changes in the boundary layer coupled with small perturbations on the external

flow. Messiter's analysis shows that the discontinuity in the pressure gradient predicted by the inviscid flow theory can be removed by using a triple deck formulation and continuous expressions for the pressure gradient can be obtained which are presumed to be correct asymptotic representations as the viscosity coefficient approaches zero. This is achieved by noting that, locally, the most important changes in the profile shape occur in a thin sublayer [11, 12] close to the wall where the changes in the viscous, pressure, and inertia forces are all of the same order as the characteristic Reynolds number tends to infinity. The remainder of the boundary layer experiences primarily a displacement effect because of the small acceleration of the fluid in the sublayer, and the resulting small decrease in the flow deflection angle is nearly constant across most of the boundary layer. The interaction of the boundary layer with the external flow occurs in a streamwise distance, $X = O(Re^{-3/8})$, and the sublayer thickness is given by $Y = O(Re^{-5/8})$, while the pressure change is found to be of $O(Re^{-3/8})$. The present sphere/cone problem is more complex due to the axisymmetric nature of the body and the fact that the approaching boundary layer at the juncture point is not that due to a flat plate as it was in Messiter and Hu's analysis. However, an approximate calculation can be

performed using their analysis to determine the scale within which the viscous smoothing takes place at the sphere/cone juncture point. This can be done by using the local Reynolds number at the sphere/cone juncture point. Figure (2) shows the results of such an analysis where the surface pressure is shown against the distance in physical coordinates. The asymptotic smoothing of the inviscid pressure gradient at the juncture point is seen to be achieved for this case in a very small physical distance upstream and, comparatively larger, but yet small distance downstream.

(c) Viscous Shock Layer

These results imply two important points. First, near the sphere/cone juncture region the correct asymptotic solution can be obtained provided the viscous set of gas dynamic equations are such that they retain the boundary layer and inertia terms in the viscous region and allow for displacement interaction with the inviscid flow. One way to ensure this criteria is to use the full Navier-Stokes equations. However, the full viscous shock layer equations also seem to be sufficient since they contain all the viscous terms of the triple deck model plus the inertia terms that take into account interaction effects in the inviscid flow.

The second important point is that the interaction effects will be significant in only a very small region of the physical flow and will be difficult to detect for high Reynolds number cases.

However, note that the present choice of the coordinate system would introduce discontinuities in the longitudinal flow gradients at the sphere/cone juncture point, as observed for the inviscid flow. It can be shown also that when viscous effects, as included in the full shock layer equations, are accounted for, the flow variables themselves are continuous through the sphere/cone juncture point (see Appendix A). This can be done by considering the integral form of the viscous equations and by evaluating them for an infinitesimally small element in the surface coordinate system (Appendix A). Note that once again, the set of equations (A7 , 14, 20) give either a trivial solution yielding $p_1 = p_2$ and $u_1 = u_2$ or a shock like jump discontinuity. It is observed that since there is no physical event that could cause a shock at the sphere/cone juncture point, the trivial solution is the only possibility indicating that the flow variables are continuous at the juncture point for the full shock layer equations in the surface coordinate system. However the same conclusion does not apply to the flow derivatives with respect to the surface distance. This is evident from

the differential form of the full shock layer equations. These full shock layer equations (1a-h) can be used to determine the jump conditions on the two sides of the sphere/cone juncture point, noting that the surface curvature, κ , takes a value of one on the spherical part and a value of zero on the conical part and also that the flow properties are continuous through this juncture point. This procedure is similar to that adopted for the inviscid set of equations. Note also that the jump conditions associated with the shock shape derivatives would remain the same as those for the inviscid case. Thus a proper physical behavior of the full viscous shock layer equations at the sphere/cone juncture point is summarized as follows:

1. The flow variables are continuous at the sphere/cone juncture point.
2. The use of a surface coordinate system introduces discontinuities in the flow gradients relative to surface distance everywhere across the shock layer at the juncture point.
3. Independent of the choice of the coordinate system, inviscid theory predicts discontinuities in flow gradients only at the surface at the sphere/cone juncture point. However the viscous

flow analysis of Messiter [7] indicates that in the limiting case of very high Reynolds number, this discontinuity would be smoothed out by the sublayer interaction effect within the inner scale length.

4. Within the viscous layer the gradient discontinuities due to the choice of the coordinate system would tend to drop out of the lead order viscous equations as the Reynolds number tends to infinity.

3. Thin Layer Analysis

Many studies [13, 14] in the past have used the thin layer version of the full shock layer equations to predict flow properties within the shock layer region for analytic bodies such as spheres, paraboloids and hyperboloids at high Mach number. The simplifying assumptions inherent in the thin shock layer approximations cause a change in the character of the governing equations and thus of the juncture point analysis presented above. In order to analyze this set of equations, the inviscid form of these equations are first considered here. Attention is first drawn to the characteristics of these equations. In the surface coordinate system the inviscid thin shock layer equations are given by equations (17, 18, 20, 21) and the normal momentum equation is given as

$$p_n(1+\kappa n) - \kappa \rho u^2 = 0 \quad (31)$$

These sets of equations can be shown to be parabolic in nature indicating that the characteristics of the flow are perpendicular to the surface of the body (see Appendix C). This suggests that for the thin shock layer equations information from the body surface is propagated along a line perpendicular to the body surface unlike the full shock layer equations where the characteristic lines are inclined at the local Mach angle of the flow. For this reason it is obvious that any discontinuity in the flow derivatives or otherwise discontinuity at the sphere/cone juncture would be felt all across the shock layer immediately at the juncture point. This physical behavior of the inviscid thin shock layer equations is significantly different from their full shock layer counterpart* and would be expected to manifest itself rather dramatically in the solutions obtained.

In order to further study the behavior of the thin shock layer equations at the sphere/cone juncture point

* It can also be shown from the analysis of the characteristics of the full shock layer equations that their characteristics tend to become perpendicular to the body surface in the limit as $\gamma \rightarrow 1$, i.e. the thin shock layer approximation is approached (see Appendix C).

attention is now directed to the normal momentum equation given above. This shows that the pressure will be a constant across the shock layer on the conical portion of the body whereas a pressure variation will be encountered on the spherical portion. This can only occur if a jump is allowed in the pressure at the sphere/cone tangency point. It is to be noted here that this need for a discontinuity in pressure is valid for inviscid as well as viscous flows since the normal momentum equation under the thin layer approximations remains unaltered. However it is of interest to note that since under the inviscid thin layer approximation the information is propagated normal to the body surface, the bow shock wave is expected to feel the presence of the sphere/cone juncture point and its attendant pressure discontinuity immediately above the juncture location.

It is, thus, important to the whole structure of the flow that the nature of the thin layer solutions be delineated. To determine the thin layer "jump condition", the approach taken here is to revert to the integral form of the full governing equations, and to assess the influence of the thin layer approximations on the generalized jump conditions so obtained. To do this one must first identify the "thin" layer terms in the general

analysis, identify their contributions to the integral formation of the governing equation, and then make the thin layer assumption. In a manner similar to the full shock layer equations, an element of infinitesimally small size is considered in this coordinate system as shown in Appendix (A). The integral form of the momentum equation (A11) when evaluated for this element, is shown to be equation (A12) when no approximation is made. It is now necessary to neglect from this equation those terms which lead to thin shock layer approximations. However, at this point those terms which are neglected in making these assumptions are unknown. Therefore, one must extend this derivation to obtain the differential form of the governing equations and track back those terms which are neglected when thin shock layer assumptions are made. This is achieved in Appendix (A). The equations, so obtained, are given as:

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \quad (32)$$

$$\rho_1 u_1 = \rho_2 u_2 \quad (33)$$

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2} \quad (34)$$

Note that these equations differ from the corresponding equations for the full shock layer form only in the one respect that the v component of velocity does not appear in the present form and thus is unrestricted in its jump behavior across the juncture point.

The equations (32-34) present the set of conditions that must be met at the sphere/cone juncture by all the flow variables in order to accommodate the pressure jump predicted by the normal momentum equation. These equations are quite similar to the "shock discontinuities" of a perfect gas except that they do not contain the v -component of velocity. The fact the v -component of velocity does not appear in these jump conditions is not surprising since a thin shock layer approximation tacitly assumes the v -component of velocity to be small compared to the longitudinal velocity and as such plays a secondary role in the conservation laws.

The admissibility of jump conditions in the flow variables at the juncture point renders the physics of the flow rather complex in this region. At the bow shock, such jump conditions would be expected to produce a discontinuous shock slope at the juncture point. It would appear that the juncture discontinuity propagates normal to the body surface through the local characteristics directly to the bow shock shape.

It is not really clear how the discontinuities in the flow variables themselves are accommodated by the thin shock layer equations. Surely discontinuities in the gradients are to be expected but more confusing is the anticipated behavior of the viscous flow regions. Here it is not clear whether discontinuous solutions can exist since preliminary analysis would seem to indicate that they would be of the subsonic "expansion shock" type. Further study of this point is warranted but shall be deferred from the current effort.

IV. NUMERICAL METHOD

1. General Considerations

Several methods have been presented for solving the "thin" shock layer version of the more general viscous shock layer equations [13, 14]. These approaches have two limitations. First they are based on the assumption that the pressure gradient normal to the body surface is established entirely by centrifugal effects, and second, that the shock wave lies parallel to the body surface. In an attempt to remove these limitations, methods have been developed [2, 15] for addressing the full shock layer equations through a relaxation process wherein the thin shock layer assumptions are removed by an iterative process. While, in general, such methods have been successful, they encounter difficulty whenever the shock layer thickness becomes large. This difficulty usually manifests itself as a divergent behavior in the iteration scheme. In an attempt to overcome this problem, a new relaxation scheme was developed in Reference [16] where an initial solution was relaxed in an artificial time like manner toward the sought after "steady state" solution. In this sense the approach is similar to the relaxation scheme presented by Davis [2], Davis and Nei [17] and Srivastava, Werle and Davis [18], the primary difference

being the manner in which the "new guess" on the solution is defined after a given step in the relaxation process. The method so developed has been found to work well for analytic bodies such as paraboloids, hyperboloids and spheres.

Application of this approach to nonanalytic bodies such as spherically blunted cones encounters numerical difficulties at a sphere/cone juncture point where the longitudinal flow derivatives in a surface coordinate system undergo discontinuous changes. The method of solution presented here represents an adaptation of the earlier time like relaxation scheme [16] to problems with imbedded discontinuities in the flow derivatives.

In order to demonstrate the present approach, the s-momentum equation of the viscous shock layer equations is first rewritten in the form

$$\frac{\partial^2 \bar{u}}{\partial \eta^2} + \beta_1 \frac{\partial \bar{u}}{\partial \eta} + \beta_2 \frac{d^2 R}{ds^2} + \beta_3 \frac{dR}{ds} + \beta_4 + \beta_5 \frac{\partial \bar{u}}{\partial \xi} = 0 \quad (35)$$

where $\beta_1, \beta_2, \beta_3, \beta_4$ and β_5 can be obtained from Reference [16] and are given in Appendix (D).

The present time like relaxation scheme utilizes a two step process in which the first step of the method is somewhat similar to an alternating direction implicit method and it yields the flow variables in the shock layer

region while the second step is used to update the shock shape itself. This scheme can be demonstrated through the s-momentum equation (7) written in a two step time formulation (see Fig. 3) as,

First Sweep *

$$\frac{\partial^2 \bar{u}^*}{\partial \eta^2} + \beta_1^* \frac{\partial \bar{u}}{\partial \eta} + \beta_2^* \left[\frac{\partial^2 R^n}{\partial s^2} - \frac{\partial R^*}{\partial t} \right] + \beta_3^* \frac{\partial R^n}{\partial s} + \beta_4^* + \beta_5^* \frac{\partial \bar{u}^*}{\partial \xi} = 0 \quad (36a)$$

Second Sweep n+1

$$\begin{aligned} \beta_2^* \frac{\partial^2 R^{n+1}}{\partial s^2} - \beta_2^* \frac{\partial R^{n+1}}{\partial t} + \beta_3^* \frac{\partial R^{n+1}}{\partial s} + \left[\frac{\partial^2 \bar{u}}{\partial \eta^2} + \beta_1 \frac{\partial \bar{u}}{\partial \eta} \right. \\ \left. + \beta_5 \frac{\partial \bar{u}}{\partial \xi} + \beta_4 \right]^* = 0 \end{aligned} \quad (36b)$$

Note that the "steady state" version of these equations are precisely the "full" shock layer equations.

The boundary conditions associated with star sweep equation (36a) are typical no slip conditions (2a, 2b) at the surface and Rankine-Hugoniot conditions (3a-3g) at the shock location. However, the boundary conditions associated with the final sweep are the same as those used in Reference [16] and are given as,

$$i) \quad \text{At } s = 0 \quad R = 0 \quad (37a)$$

$$ii) \quad \text{At } s = s_{\max} \quad R_{\max}^{n+1} = R_{\max}^* \quad (37b)$$

There are two points of interest produced by the sphere/cone curvature discontinuity. First note that the coefficients in the s-momentum equation (7) contain the surface curvature, which itself undergoes a discontinuous change from a value of one on the spherical body to a value of zero on the conical body at the sphere/cone juncture point. This then causes the β coefficients of equation (36) to undergo discontinuous changes at the juncture point. As a second point of interest, it is also noted that the derivatives of the shock shape with respect to s , explicitly appear in the governing equations, such as in equation (36). These shock derivatives have been shown to undergo discontinuities at the juncture point in Section III. This jump condition on the first derivative of the shock shape is given by equation (26). A similar jump condition on the second derivative of the shock shape can be obtained through use of the governing differential equations.

As a result then it is seen that the governing equation (36) contains both flow coefficients and shock derivatives which undergo discontinuous change at the juncture point which necessarily produce solutions with discontinuous gradients (see Section III). Such results obviously require modifications in order to obtain numerical solutions of this set of governing equations

to assure that finite differencing is not done across such discontinuous regions. Note that the numerical difficulties associated with equation (36a) can be overcome by structuring the finite difference grid system with a point at the juncture thereby avoiding differencing across the discontinuity. However difficulty is still encountered in the second step of the solution process due to the discontinuities that occur in the shock shape derivatives. The occurrence of these discontinuities requires the use of special difference relations at the juncture point. It will be shown here, through a model problem representing the second step of the present numerical scheme that this difficulty can be overcome if the difference form of the differentials are formulated such that they comprehend the juncture jump conditions.

2. A Model Problem

The governing equation for a model problem and its associated boundary and jump conditions are formulated here in order to demonstrate the concepts associated with the viscous shock layer solution for spherically blunted cones.

The governing equation for this model problem is taken to be analogous to the second step of the viscous shock layer scheme and is given as

$$\frac{d^2 R}{ds^2} + \alpha_1 \frac{dR}{ds} + \alpha_2 R + \alpha_3 = 0 \quad (38)$$

Note that the coefficients α_1 , α_2 , α_3 in the above equation are to be selected so that this equation models the second step of the viscous shock layer scheme. This would require that either some or all of these coefficients undergo a discontinuous change at the model juncture point in the solution region. For present purposes the coefficients are taken to be one set of constants in the region ahead of the juncture and a different set of constants aft of the juncture. A comparison with the second step of the viscous shock layer scheme shows that the coefficient α_2 does not encounter any jump whereas α_1 and α_3 do encounter jumps in their magnitudes at the sphere/cone tangency point. Thus, the present model problem is set up such that the coefficients α_1 , α_2 , α_3 take constant values (corresponding to the spherical section) in one region and different constant values (corresponding to the conical section) aft of the juncture location.

The boundary conditions to be applied to this model problem are established to closely correspond to the second step of the viscous shock layer solution. These then are given as

$$R = 0 \quad \text{at} \quad s = 0 \quad (39a)$$

$$R = R_\ell \quad \text{at} \quad s = s_{\max} \quad (39b)$$

In addition to these, jump conditions analogous to those discussed for the viscous shock layer equations must be established and applied at the juncture point.

At the juncture it is required that the function R , be continuous but that the first derivative, dR/ds , be represented by the relation

$$\left(\frac{dR}{ds}\right)_{-s_{\text{jump}}} = K_1 \left(\frac{dR}{ds}\right)_{+s_{\text{jump}}} \quad (40)$$

Note that the value of K_1 will be determined here to match the jump actually encountered by the viscous shock layer shock derivative at the sphere/cone tangency point.

From the governing equation (38), it is found that the second derivative, d^2R/ds^2 , also undergoes a jump at the juncture point as given by the relation

$$\left(\frac{d^2R}{ds^2}\right)_{-s_{\text{jump}}} = \left(\frac{d^2R}{ds^2}\right)_{+s_{\text{jump}}} + K_2 \left(\frac{dR}{ds}\right)_{+s_{\text{jump}}} + K_3 \quad (41a)$$

where

$$\begin{aligned} K_2 &= (\alpha_1)_{+s_{\text{jump}}} - K_1 (\alpha_1)_{-s_{\text{jump}}} \\ K_3 &= (\alpha_3)_{-s_{\text{jump}}} - (\alpha_3)_{+s_{\text{jump}}} \end{aligned} \quad (41b)$$

The exact solution for the above equations, associated boundary conditions and jump requirements can be found easily and is given as

$$R = A e^{ms} + B e^{-ns} - \alpha_3/\alpha_2$$

for either region, where

$$m = -\alpha_1/2 + \sqrt{\alpha_1^2/4 - \alpha_2} > 0 \quad \text{for} \quad \alpha_2 < 0$$

$$n = \alpha_1/2 + \sqrt{\alpha_1^2/4 - \alpha_2} > 0 \quad \text{for} \quad \alpha_2 < 0$$

This gives,

$$R = (\alpha_3/\alpha_2) e^{ms} + B[e^{-ns} - e^{ms}] - \alpha_3/\alpha_2 \quad (42)$$

when $0 \leq s \leq -s_{\text{jump}}$

Also,

$$R = R_l e^{D_1(s-s_{\text{max}})} - B_1[e^{D_1(s-2s_{\text{max}})} - e^{-D_1s} + \alpha_3/\alpha_2 [e^{D_1(s-s_{\text{max}})} - 1]] \quad (43)$$

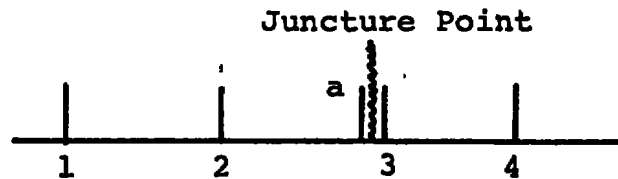
for $+s_{\text{jump}} \leq s \leq s_{\text{max}}$

where $D_1 = \sqrt{-\alpha_2}$ for $\alpha_1 = 0$ in this region. Equations (42, 43) contain two undetermined constants B and B_1 which can be determined by using the jump condition (40) and the condition that R be continuous at the juncture point.

3. Model Problem Finite Difference Formulation

The finite difference form of the differentials in equation (38) requires special attention at the juncture point in order to account for the jump conditions associated with the various derivatives. This formulation is shown through the figure below where typical mesh points are

shown with a jump occurring immediately ahead of point 3.



Mesh point "a" is located immediately ahead of the juncture point. While formulating the difference form of the derivatives at point "3", a Taylor's series representation which avoids any series expansion across the discontinuity is utilized. For this case, then, a Taylor's series expansion is used from point 3 to 4 and from point "a" to 2. Points "a" and 3 across the discontinuity are related through the jump conditions (40, 41). This procedure yields after proper manipulation (see Appendix E) the following forms of the difference representations of the differentials in equation (38) at the juncture point.

$$\left(\frac{dR}{ds}\right)_3 = \frac{R_4 - R_2}{\Delta(1+K_1 - \frac{\Delta}{2} K_2)} + \frac{\Delta K_3}{2(1+K_1 - \frac{\Delta}{2} K_2)} + O(\Delta^2) \quad (44a)$$

$$\left(\frac{d^2R}{ds^2}\right)_3 = \frac{(R_2 - R_3) + (K_1 - \frac{\Delta}{2} K_2)(R_4 - R_3)}{(\Delta^2/2)(1+K_1 - \frac{\Delta}{2} K_2)} - \frac{2K_3}{\Delta^2(1+K_1 - \frac{\Delta}{2} K_2)} + O(\Delta) \quad (44b)$$

Note is made here that these expressions contain the constants associated with the jump conditions (40, 41) and that they reduce to their proper central difference representation for zero strength jump (e.g., $K_1=1$, $K_2=0$ and $K_3=0$). A formally second order accurate representation can similarly be formulated for the second derivative by evaluating the

error term of equation (44b) using the governing differential equation at the point where the jump is taking place in the solution regime. The error term in Equation (44b) is given as

$$E = \frac{\Delta}{3} \left[\frac{(K_1 - \frac{\Delta}{2} K_2) R_3'''' - R_a''''}{(1 + K_1 - \frac{\Delta}{2} K_2)} \right] + O(\Delta^2)$$

For the model problem the first term (order Δ) of the error expression can be evaluated by first differentiating the governing differential equation and then evaluating it on the two sides of the point where the jump in derivatives occurs. This yields

$$R_a'''' + \alpha_{1a} R_a''' + \alpha_{2a} R_a'' = 0 \quad (45a)$$

and

$$R_3'''' + \alpha_{13} R_3''' + \alpha_{23} R_3'' = 0 \quad (45b)$$

Manipulation of equations (45a,b) results in

$$\begin{aligned} (K_1 - \frac{\Delta}{2} K_2) R_3'''' - R_a'''' &= -(K_1 - \frac{\Delta}{2} K_2) \alpha_{13} R_3''' \\ &+ \alpha_{1a} R_a''' + \alpha_{2a} R_a'' - (K_1 - \frac{\Delta}{2} K_2) \alpha_{23} R_3'' \end{aligned} \quad (46)$$

A substitution of this expression in (44b) would result in a second order accurate second derivative at the juncture point. For further details see Appendix (E).

For the sake of identification in the rest of this text, the first of these above difference expressions (44) will be referred to as the "first order accurate scheme", even though only the second derivative at the juncture point experiences a formally first order error. The second of these where the second derivative is also formally second order accurate will be referred to, here, as the "second order accurate" scheme.

Before the numerical scheme outlined above is used in the viscous shock layer equations, a test of this scheme's ability to approximate the exact solution is essential.

4. Model Problem Numerical Results

Figure 4 shows the variable "R" as a function of distance for the model problem. The coefficients α_1 , α_2 , α_3 in the model equation were chosen to approximately represent the sphere/cone juncture point of a 40° half angle spherically blunted cone. Severe error is seen in the numerical solution when the jump effects are ignored. However, when proper jump effects are accounted for, the exact solution is virtually recovered by the present numerical scheme. Figure 5 shows the first derivative for the same problem. It is noted that the discontinuity predicted by the exact solution is virtually captured exactly by the numerical scheme if proper jump

effects are accounted for in the numerical scheme. Figure 6 shows the second derivative, d^2R/ds^2 , as a function of surface distance. It is seen also from this figure that large numerical errors are present when the effects of the jump are ignored. However, the exact solution is accurately recovered when proper jump effects are included.

For the sake of completeness, the model problem was also solved using the "second order accurate scheme" of equation (44). It was found that for this case the numerical difference could not be detected to the scale of plot shown in Figures 4 through 6. In order to clarify this point, further studies were undertaken. Figure 7 shows the model problem shock curvature, d^2R/ds^2 , at the junction point as a function of the step size " Δs " for the "first order" and "second order" numerical schemes. Note here is made of the fact that the so-called "first order scheme" employs a formally first order accurate representation of the second derivative at only one point in the mesh system, i.e. at the juncture point. It is seen from Figure 7 that the "second order scheme" shows a parabolic behavior as the step size " Δs " is reduced, as one would expect. However, the "first order scheme" does not show a linear dependence on Δs but rather a parabolic behavior. To further detail these results, Figure 8 replots

these curves against the square of the step size " Δs ". This figure clearly indicates that both the "first" and "second" order schemes yield results that approach the exact solution as though they were second order accurate. The explanation of these results is given in detail in Appendix (F) where it is verified that it is basically a manifestation of the fact that a local truncation error of order Δs at a finite number of points in a finite difference mesh does not necessarily produce a first order global error.

It is, therefore, established that the so-called "first order accurate scheme" is essentially second order accurate. For this reason it was found unnecessary to use the "second order accurate scheme" for present purposes.

5. Application to the Full Viscous Shock Layer Equations

The finite difference expressions so developed can now be applied to the solution of the full viscous shock layer equations for hypersonic flow past spherically blunted cones. A detailed description of the method for evaluating the jump conditions associated with the shock wave derivatives at the sphere/cone juncture point for the full viscous shock layer equations is presented in Appendix (G). It is shown here, that the proper jump condition associated with the first derivative, dR/ds ,

at the juncture point can be estimated by purely geometrical considerations and the fact that the shock wave shape, R , itself is smooth at this point. However, the jump condition associated with the second derivative, d^2R/ds^2 , must be obtained by using the differential equation itself on the two sides of the sphere/cone tangency point in a manner similar to that adopted for the model problem discussed earlier. This is shown further in Appendix (G). It is shown in this appendix that care must be used while evaluating the jump conditions in the viscous shock layer code. For present purposes, the jump conditions were evaluated at the first mesh point away from the wall.

6. Overall Method of Solution

The overall method of solution for the full viscous shock layer equations is as follows. An initial guess was first made on the shock shape. Based on this guess the first and second derivatives of the shock distance were computed using central differences at points away from the sphere/cone tangency point. However, since jump conditions on the shock derivatives are not known initially at the sphere/cone juncture point, a second order accurate three point backward difference schemes was used on the spherical part and a three point forward difference scheme was used on the conical part for the first

derivative of the shock stand off distance in order to avoid any differencing across the juncture point. In a like manner, four point second order accurate schemes were used for the second derivative of the shock stand off distance at the juncture point. The star sweep equations were then solved by starting at the stagnation point, where both $\partial \bar{u} / \partial \xi$ and $\partial \bar{t} / \partial \xi$ vanish, thus reducing the governing equations to ordinary differential equations. The first equation solved was the energy equation so that thereafter all quantities such as viscosity related to temperature could be evaluated. Next, the s-momentum equation was integrated to determine a \bar{u} -velocity profile, and then the continuity equation was solved to determine first the shock stand-off distance from equation (10) and then the \bar{v} -component of the velocity from equation (16). Finally equation (11) was integrated to determine the local pressure level. The coefficients in the governing equations were then reevaluated using the new flow variables. Repetition of the above steps at a given station continued until the solution converged. The method then stepped along the body surface and iterated at each station to achieve converged solutions. To accelerate the convergence process, the previous station values of the profiles were used at each new step as a first guess. One difficulty encountered during this

iteration scheme was the presence of an oscillatory behavior of the normal velocity component, \bar{v} , at some station in the s-direction [Ref. 16]. This oscillatory behavior of the physical quantities was overcome by an under-relaxation scheme as shown:

$$w = F_1 w_1 + (1-F_1)w_2$$

where w_1 is the most recently calculated physical quantity and w_2 is the value obtained from the previous calculated value of this quantity. It was observed that a value of F_1 of 0.2 to 0.4 produced convergence in most cases considered. In general it was also found that such an under-relaxation technique was needed only for the pressure and v-component of velocity.

Once the above method had passed over the entire mesh the second sweep equations were invoked. The final sweep equation (36b) was then solved using the two boundary conditions of equations (37a,b). No iteration of the final sweep equation was required since it is linear. However note that the final sweep equation requires the necessary jump conditions associated with the first derivative of the shock stand off distance, dR/ds and also that associated with the second derivative, d^2R/ds^2 . These jump conditions were evaluated using the flow properties obtained in the star sweep calculations. The

final sweep equation was required since it is linear. However note that the final sweep equation requires the necessary jump conditions associated with the first derivative of the shock stand off distance, dR/ds and also that associated with the second derivative, d^2R/ds^2 . These jump conditions were evaluated using the flow properties obtained in the star sweep calculations. The shock shape obtained from the final sweep was used then to solve the next star sweep in time. The procedure continued in time until two alternate final sweeps converged to a desired degree of accuracy. Appendix (H) discusses further details of the computer program used to obtain the present numerical results.

7. Grid Sizes for Shock Layer Solution

The following normal step sizes distributions were used in the finite difference solution of the full viscous shock layer equations for the cases presented in the following section.

$Re_{\infty} = 1.515 \times 10^3$	
η_{range}	$\Delta\eta$
0.0 - 0.050	0.001
0.05 - 0.65	0.015
0.65 - 1.0	0.035

$Re_{\infty} = 3 \times 10^5$	
η_{range}	$\Delta\eta$
0.0 - 0.005	0.0001
0.005 - 0.50	0.005
0.50 - 1.0	0.0099

V. RESULTS AND DISCUSSION

The general analysis and the numerical techniques discussed earlier were used to obtain the solutions of the full viscous shock layer equations for hypersonic flow past various spherically blunted cones in order to test the reliability of this technique. Since the interest in the present study was centered on the sphere/cone tangency region, numerical solutions were generated only to about 2-3 nose radii downstream of the stagnation point for a range of large as well as small cone angles of the spherically blunted cones. Numerical solutions were obtained for a wide range of cone half angles from 30° to 0° at various test conditions corresponding to available data and other calculations.

Figure 9 shows the surface pressure distribution obtained here for a 30° half cone angle spherically blunted cone at a free stream Mach number, $M_\infty = 10$, free stream Reynolds number, $Re_\infty = 3 \times 10^5$ and a wall to stagnation temperature ratio, $T_w/T_o = 0.05$. These test conditions were chosen in order to compare the predicted numerical results with the inviscid solutions of Inouye et al. [8] for the same body. The present calculations for this case were made using a variable normal step size, Δn , which ensured at least 10-15 mesh points within the

boundary layer regime while the longitudinal step size, Δs , was selected such that a mesh point of the numerical scheme coincided with the sphere/cone juncture point. The time step size, Δt , was taken to be 3.5.

Figure 9 shows that when the shock jump conditions are accounted for in the finite difference formulation as shown earlier, the predicted surface pressure compares well with the inviscid solution. It is of interest to note here that the discontinuity in the surface pressure gradient at the sphere/cone juncture point predicted by the inviscid theory is virtually reproduced by the present viscous model for this very high Reynolds number case. However, in the light of the analysis of Messiter and Hu [7] for a simple two-dimensional flow with a curvature discontinuity, one might have anticipated a viscous smoothing of the discontinuity in the surface pressure gradient at the juncture point. It is now clear from the discussion of Section III that the viscous smoothing for this problem is of a very mild nature and occurs over such a short distance that it is not seen to the scale of the present calculations.

Figure 9 also presents two other numerical results for the same test conditions. One of these is the case where a numerical solution of the full viscous shock layer equations was obtained by ignoring the relevant

jump conditions associated with the shock derivatives at the juncture point and by adjusting the mesh of the numerical scheme such that the juncture point lies between two mesh points. This case then allows an assessment of the numerical errors that are introduced in the computational method when one ignores the relevant jump effects associated with the juncture point. Note that the numerical errors are large in the juncture region and tend to diminish away from the juncture region. The second case shown in Figure 9 is similar to the first except that the mesh system was aligned so that a mesh point coincided with the sphere/cone tangency point. This case allows an assessment of the importance of the shock jump conditions on the surface properties. Apparently these are of a dominant nature in this region of the flow.

Figure 10 shows the surface heat transfer distribution for this case under identical flow conditions. It is observed that the erratic behavior of the computational results persist when the jump effects are ignored whereas the inclusion of these effects tend to eliminate this erratic behavior completely. From equation (26) it is seen that the jump conditions associated with the shock derivatives tend to increase in magnitude as the cone angle for the spherically blunted cones is reduced. It is, therefore, pertinent to test this computational

technique for lower cone angles in order to establish the generality of this scheme. Further numerical solutions were obtained for lower cone angles ranging from 20° to 0° in order to assess the influence of the jump effects on the surface properties. Difficulties were encountered here while attempting to reduce the cone angle mainly due to the choice of the initial shock shape for such bodies. This was overcome here by reducing the cone angle in increments of about 5° with the number of mesh points between the juncture and stagnation point kept fixed. This resulted in an increase in longitudinal step size, Δs , as the cone angle was reduced but this technique was found to work well for all cases that are presented here. It should also be noted that care was exercised to at least include 10-15 points within the boundary layer while selecting the normal step size, Δn . Figure 11 through 15 present the results for such a calculation for cone angles ranging from 20° to 0° . It is seen from Figures 11 and 13, which show the surface pressure distributions, that the results of such a calculation compare well with the inviscid solution, when the proper jump conditions associated with the shock derivatives are included in the solution scheme. These figures also show the case when such jump effects are ignored in the calculation, thereby causing rather

erratic behavior. Figures 12 and 14 show similar results for the surface heat transfer rates. Note that Figure 15 for a 0° cone angle (spherical-cylinder body) does not contain the numerical results corresponding to the no jump case. This is because the errors were of such large magnitude at the juncture point that a properly converged solution could not be obtained.

Thus far it has been shown that the present computational approach yields good numerical solutions in comparison with the inviscid theory [8]. Further comparisons are now presented with the available experimental data for spherically blunted cones. Figure 16 presents surface pressure for 7.5° half angle spherically blunted cone at free stream Mach number, $M_\infty = 13.41$, free stream Reynolds number, $Re_\infty = 1515$, wall to stagnation temperature ratio, $T_w/T_o = 0.0741$ and a free stream temperature, $T_\infty = 200^\circ R$ corresponding to the experimental data of Pappas and Lee [19]. Significant differences from that of the inviscid results are noticed at the sphere/cone juncture point for this case. The inviscid pressure distributions shown earlier through Figures 11-13 predict a discontinuity in the pressure gradient at the sphere/cone juncture point. However for the present low Reynolds number of 1515, viscous effects smooth the discontinuity completely. The result of the present calculations are seen to compare well with the data shown in Figure 16.

Note also that similar numerical calculations which do not include the proper jump effects at the juncture point are seen to yield seriously erroneous results. Figure 17 presents the ratio of the wall to the stagnation point heat transfer for the same test case. The comparison of the present calculations with experimental data when the jump effects are included is seen to be excellent. Again large errors are observed when these jump effects are ignored. Figures 16 and 17 also show the numerical results obtained by Miner and Lewis [5] for the same body (with an artificially smoothed juncture point) under identical test conditions. It is seen from these figures that the present results tend to show better agreement with the experimental data when shock jump effects are directly accounted for at the sphere/cone juncture point.

VI. CONCLUSIONS AND RECOMMENDATIONS

An analysis of the physical flow behavior at the sphere/cone tangency point has been made. This analysis indicated that, independent of the choice of the coordinate system, inviscid theory would predict a discontinuity in the flow gradients only at the surface at the sphere/cone juncture point. However, following the analysis of Messiter [7], it was found that in the limiting case of very high Reynolds number this discontinuity would be smoothed out by the sublayer interaction effect within the inner scale length. It has also been shown that the use of a surface coordinate system introduces discontinuities in the flow gradients relative to surface distances everywhere across the shock layer and at the body surface at the juncture point. Within the viscous layer this gradient discontinuity due to the coordinate system would tend to vanish to lead order as the Reynolds number tends to infinity. Analytical jump conditions were developed at the sphere/cone juncture point for these discontinuous flow gradients associated with the choice of surface coordinate system. Finite difference formulations were then developed that account for these embedded gradient discontinuities in order to eliminate numerical difficulties

in the solution of the full viscous shock layer equations. Such solutions were obtained by a numerical scheme which utilizes a time dependent relaxation technique for the bow shock wave shape. Comparisons of the present results with inviscid solutions at high Reynolds numbers and experimental data at intermediate ones were found to be good.

While the present technique for treating the sphere/cone juncture region has been shown to yield good results, the present numerical scheme which is essentially a time dependent relaxation technique encountered certain difficulties worth noting. One difficulty that does arise is the oscillatory behavior observed in the iterative solution of the shock layer equation at some point downstream on the surface. While an under-relaxation scheme was found to effectively remove this problem for most flow conditions of interest, it is recommended that in future studies the continuity and normal momentum equations be coupled during the iteration process. Another difficulty that was found for this relaxation scheme was the initialization process used for the bow shock shape. While this technique enjoys a greater degree of flexibility as compared to previous techniques, non-the-less difficulties were encountered for the low cone angle cases of the spherically blunted cone studies. Future studies should consider use of the inviscid bow shock shape as the natural initial shape for this relaxation technique.

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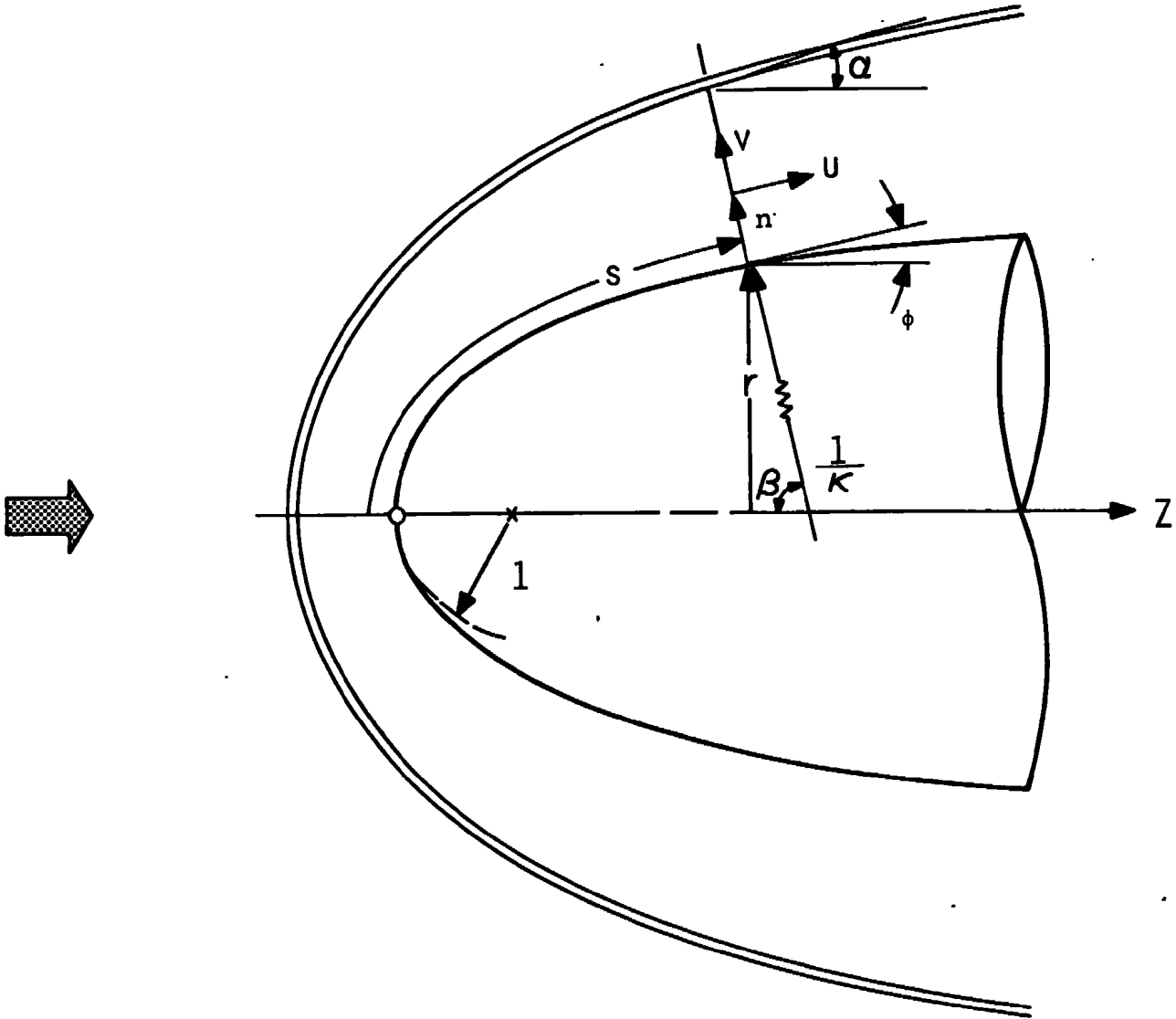


FIGURE 1. COORDINATE SYSTEM

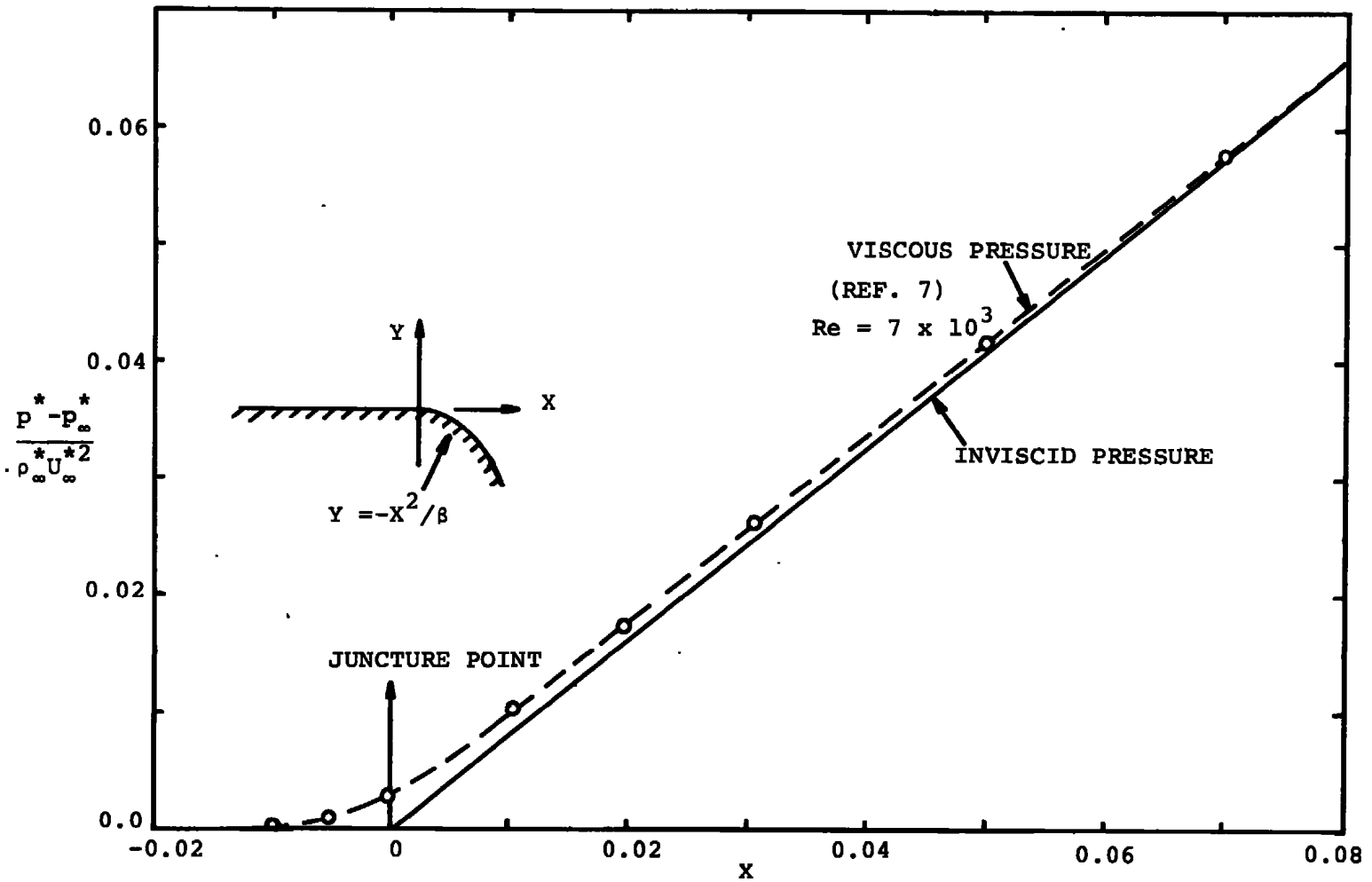


FIGURE 2 SUBLAYER INTERACTION PRESSURE DISTRIBUTION

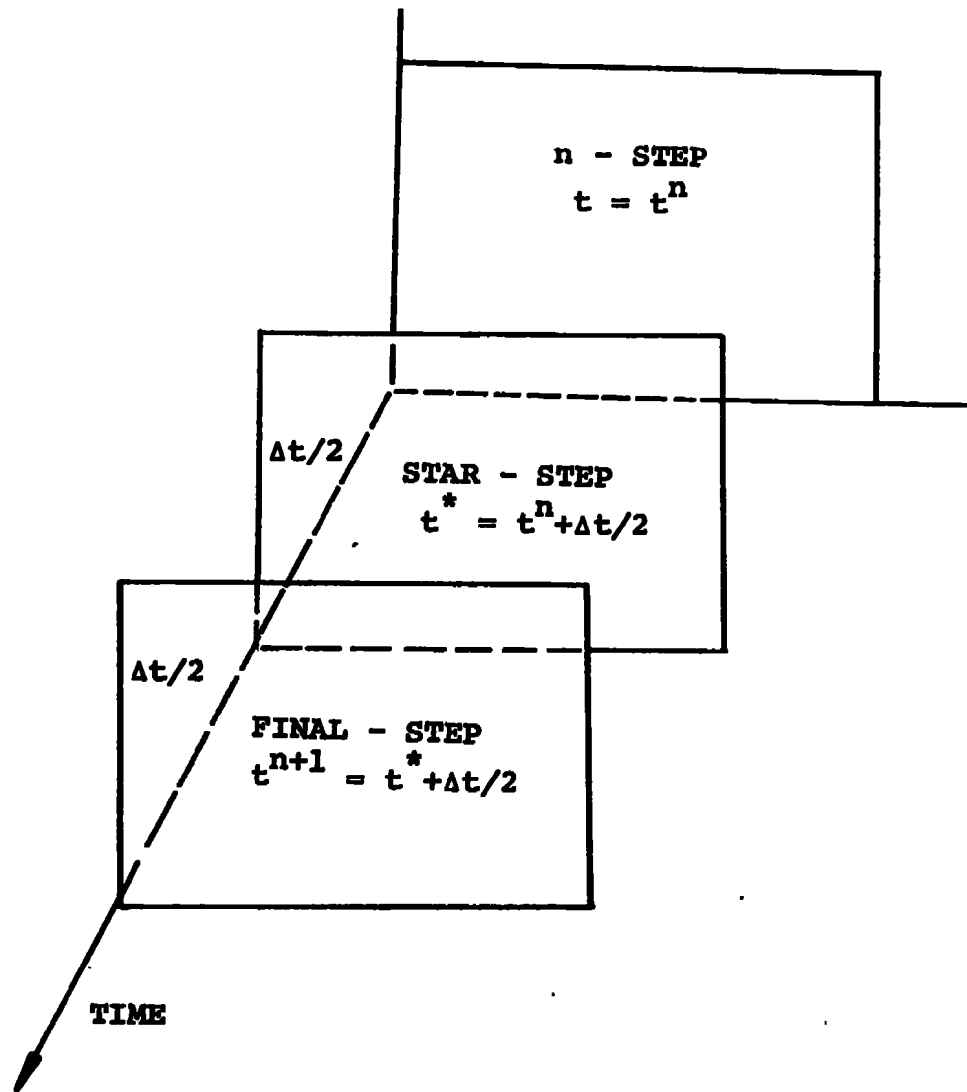


FIGURE 3. ADI NUMERICAL SCHEME

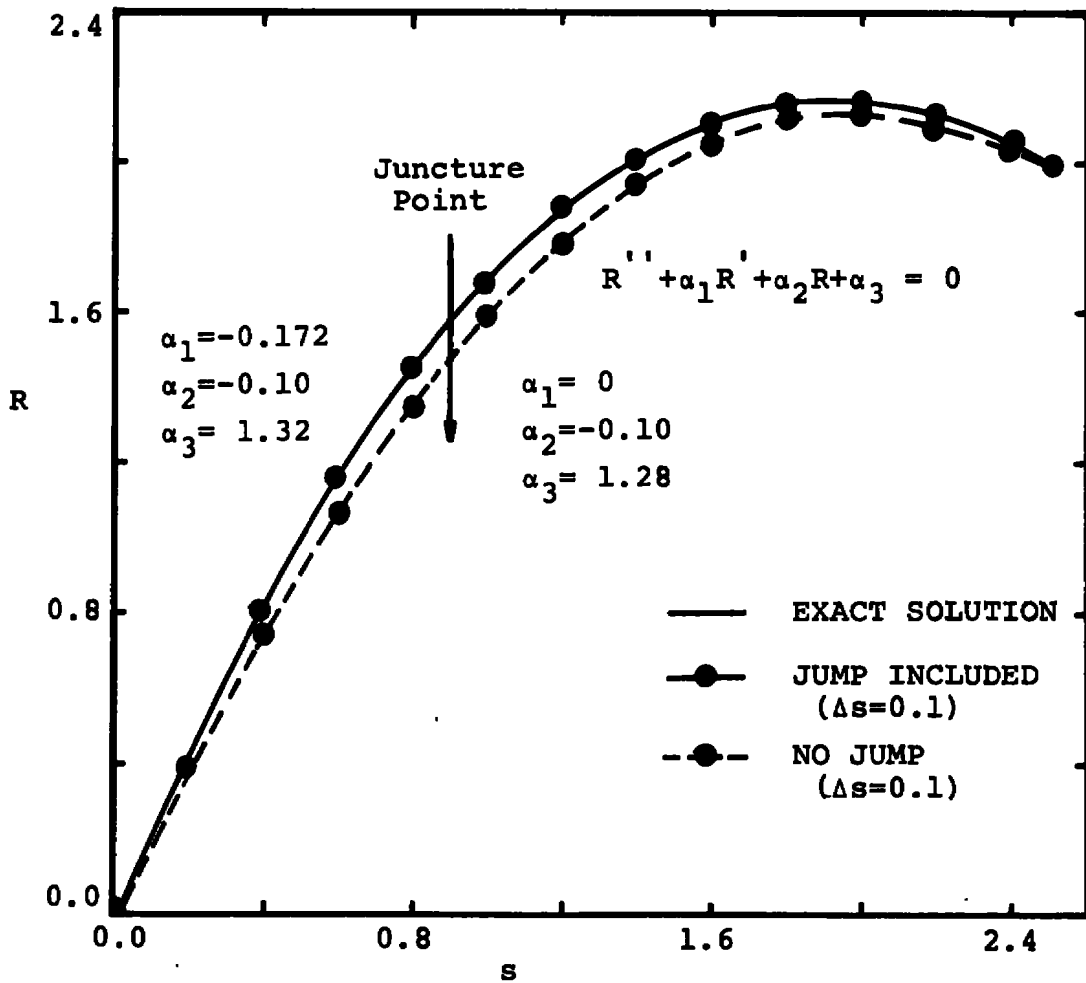


FIGURE 4 MODEL SHOCK SHAPE

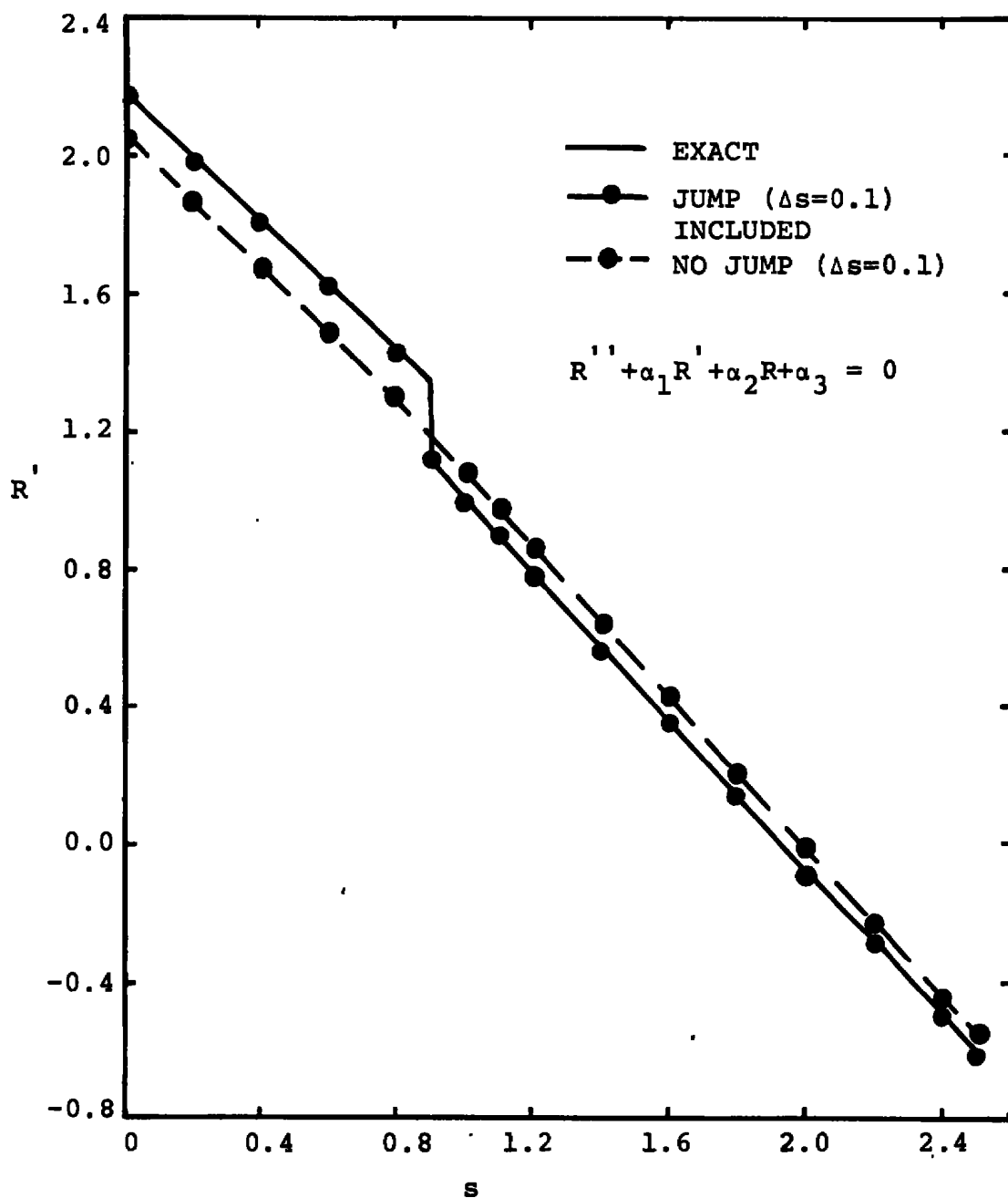


FIGURE 5 MODEL SHOCK SLOPE

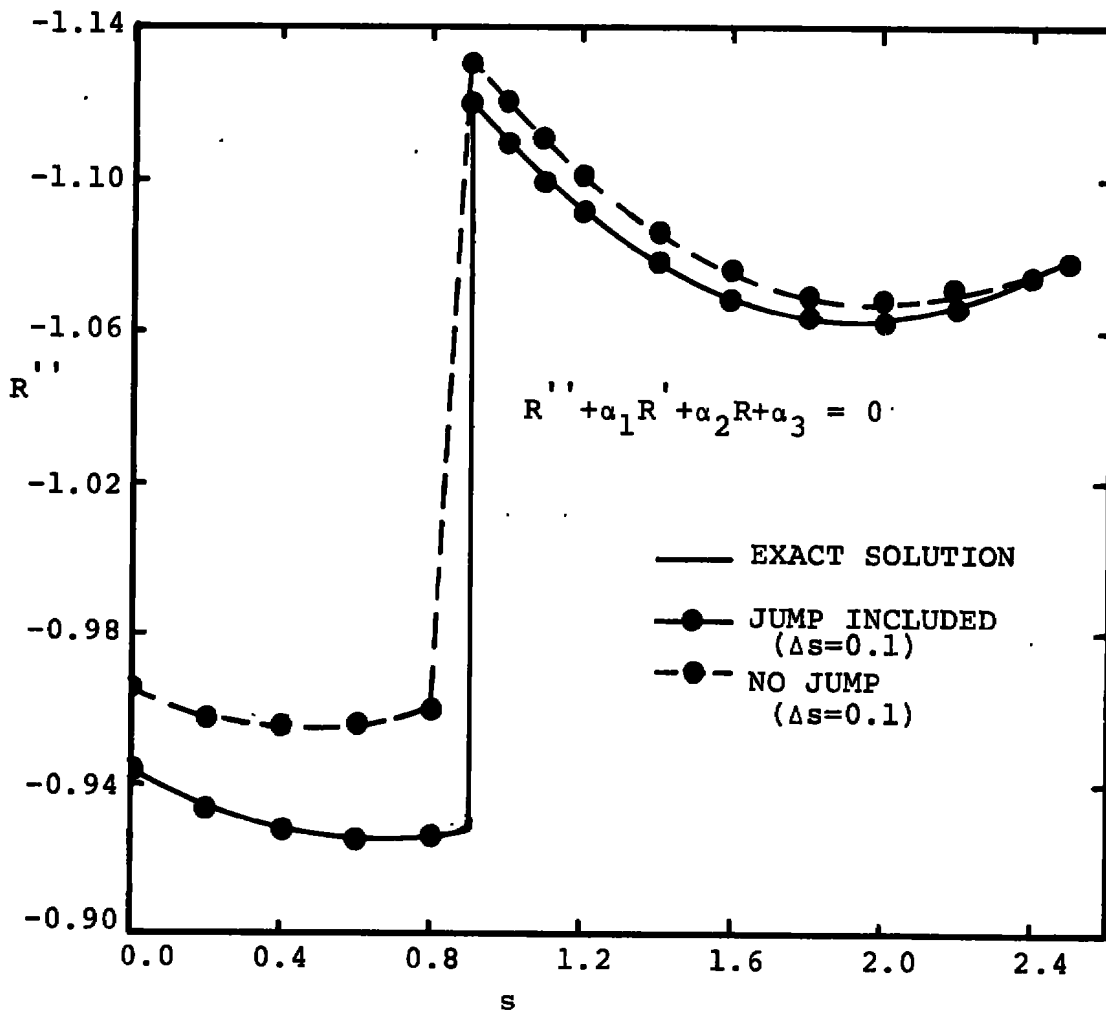


FIGURE 6 MODEL SHOCK CURVATURE

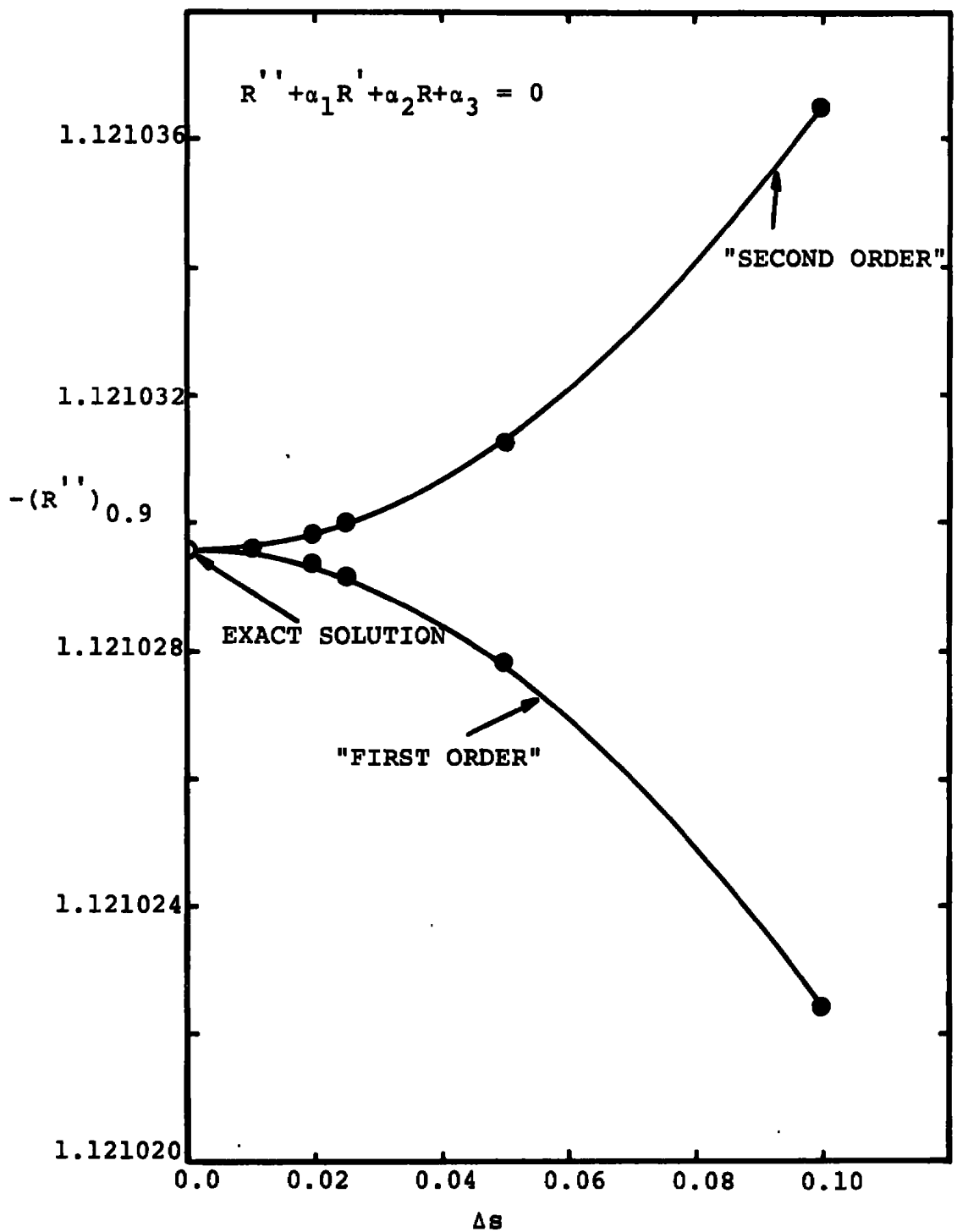


FIGURE 7 MODEL STEP SIZE STUDY

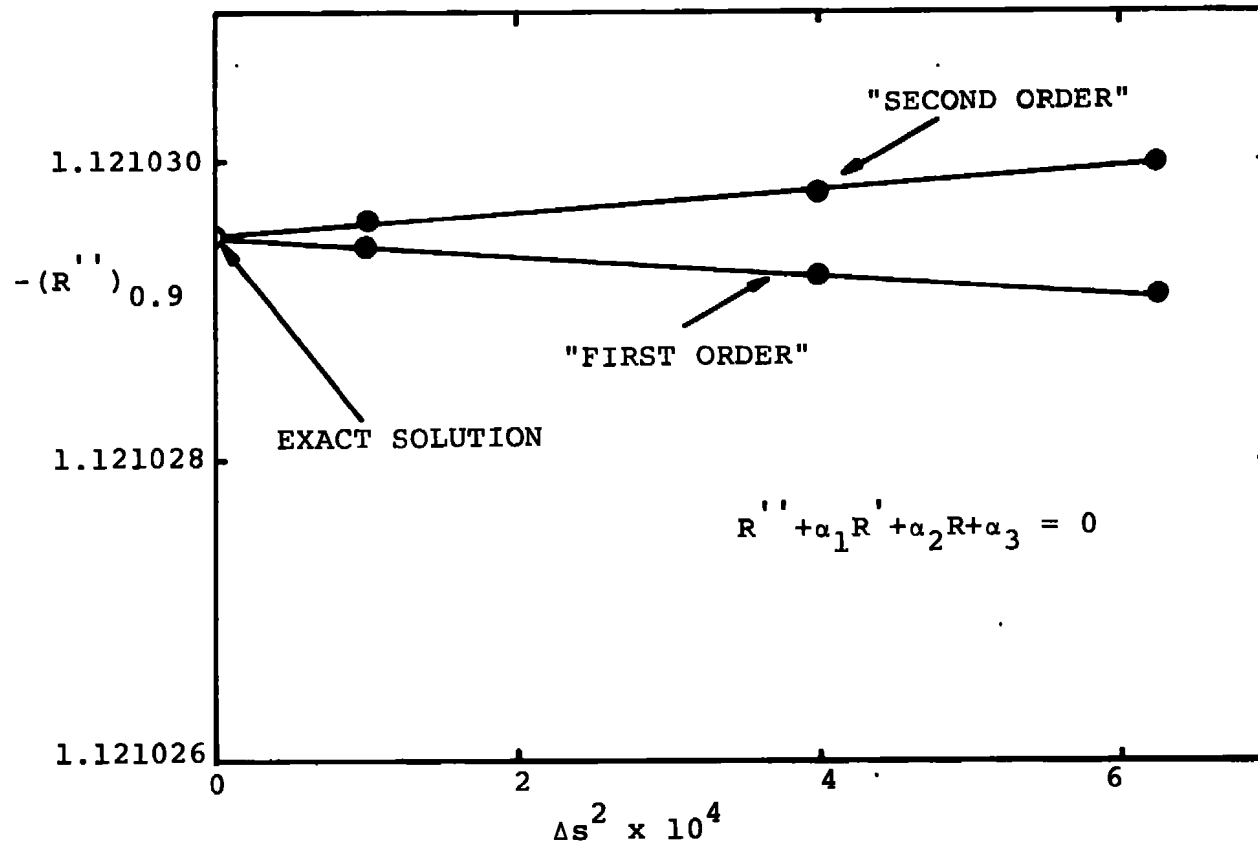


FIGURE 8 MODEL STEP SIZE STUDY

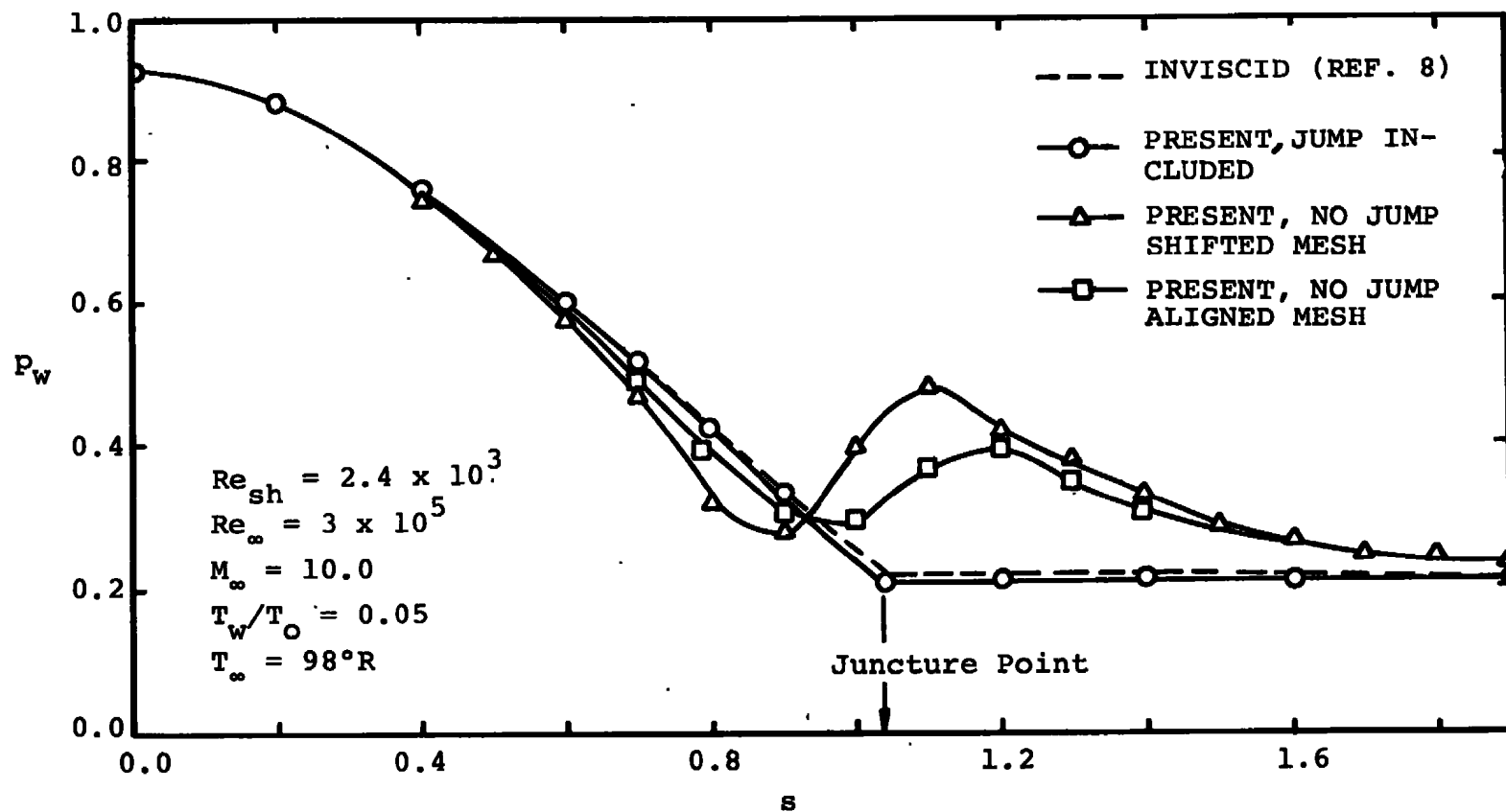
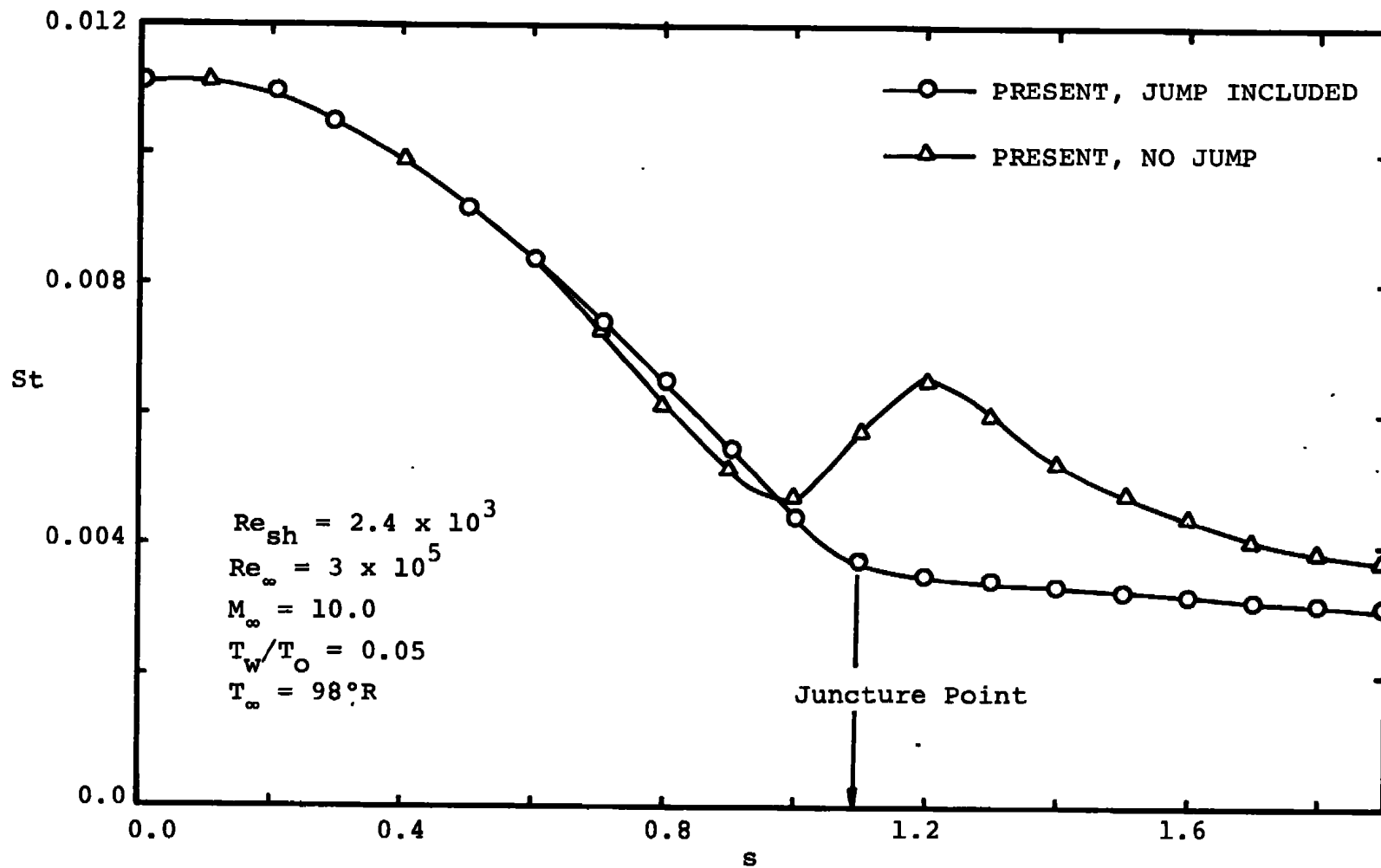


FIGURE 9 SURFACE PRESSURE DISTRIBUTION FOR A 30° HALF ANGLE SPHERE-CONE

FIGURE 10 HEAT TRANSFER DISTRIBUTION FOR A 30° SPHERE-CONE

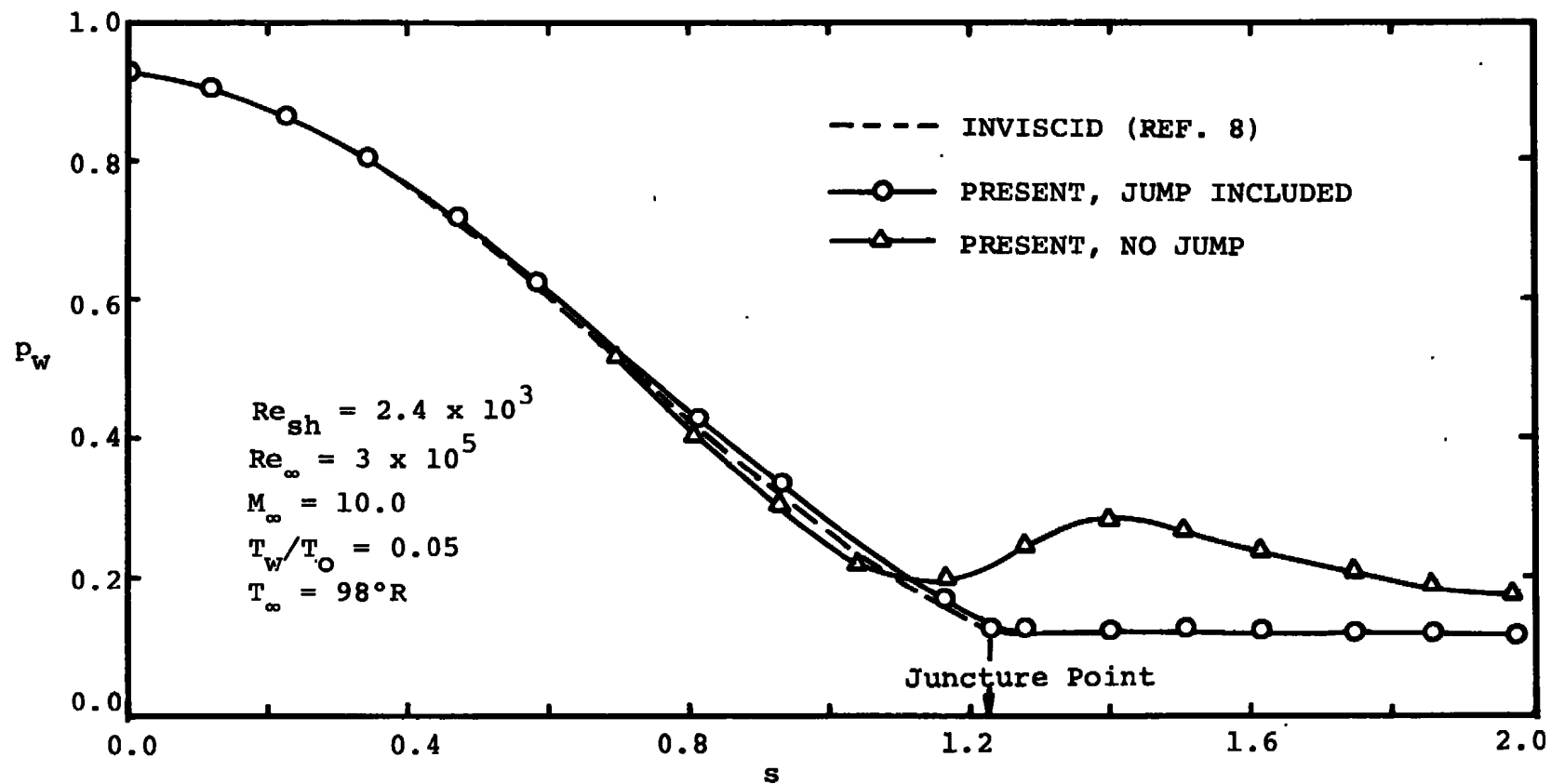


FIGURE 11 SURFACE PRESSURE DISTRIBUTION FOR A 20° SPHERE-CONE

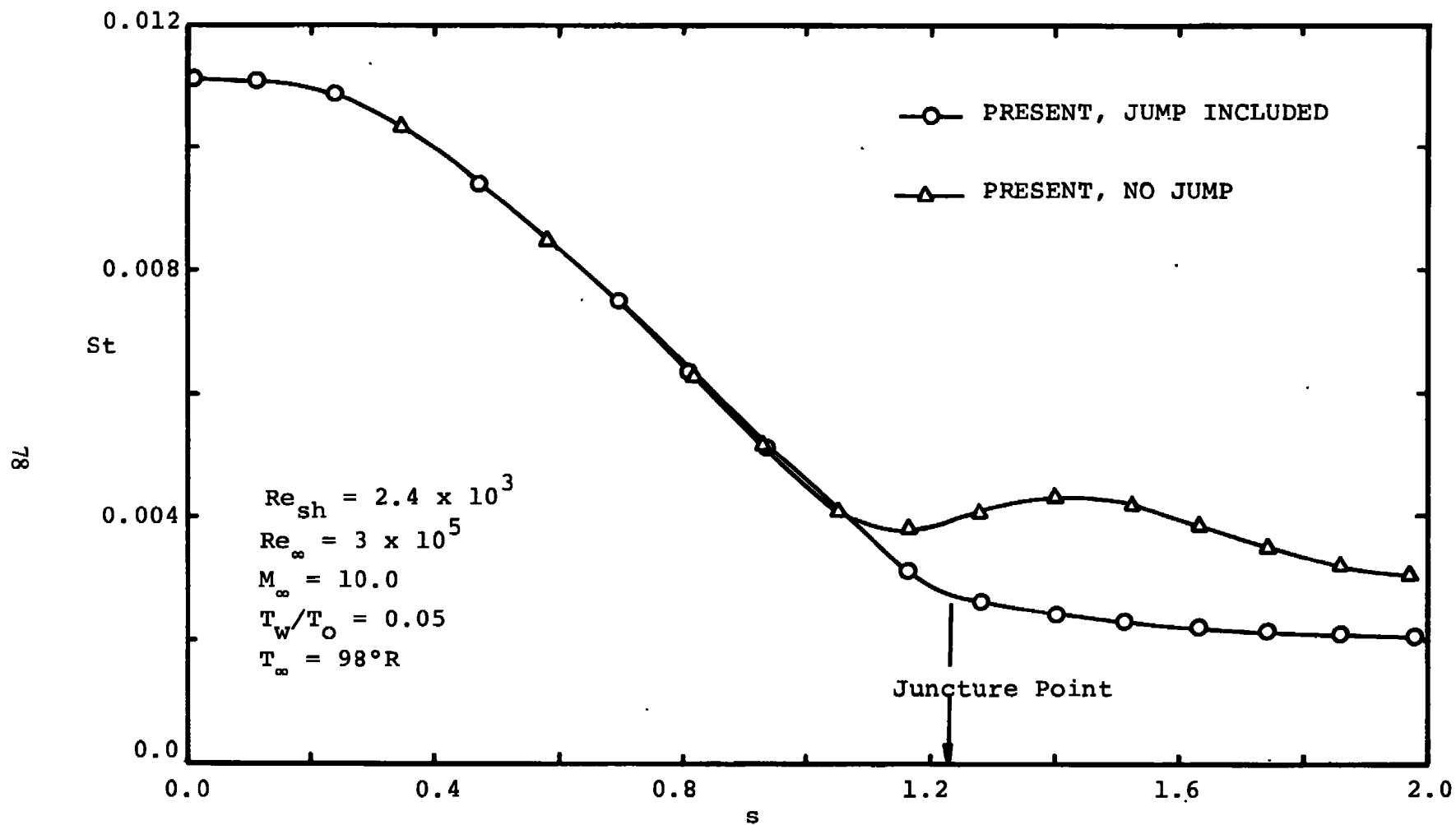


FIGURE 12 HEAT TRANSFER DISTRIBUTION FOR A 20° SPHERE-CONE

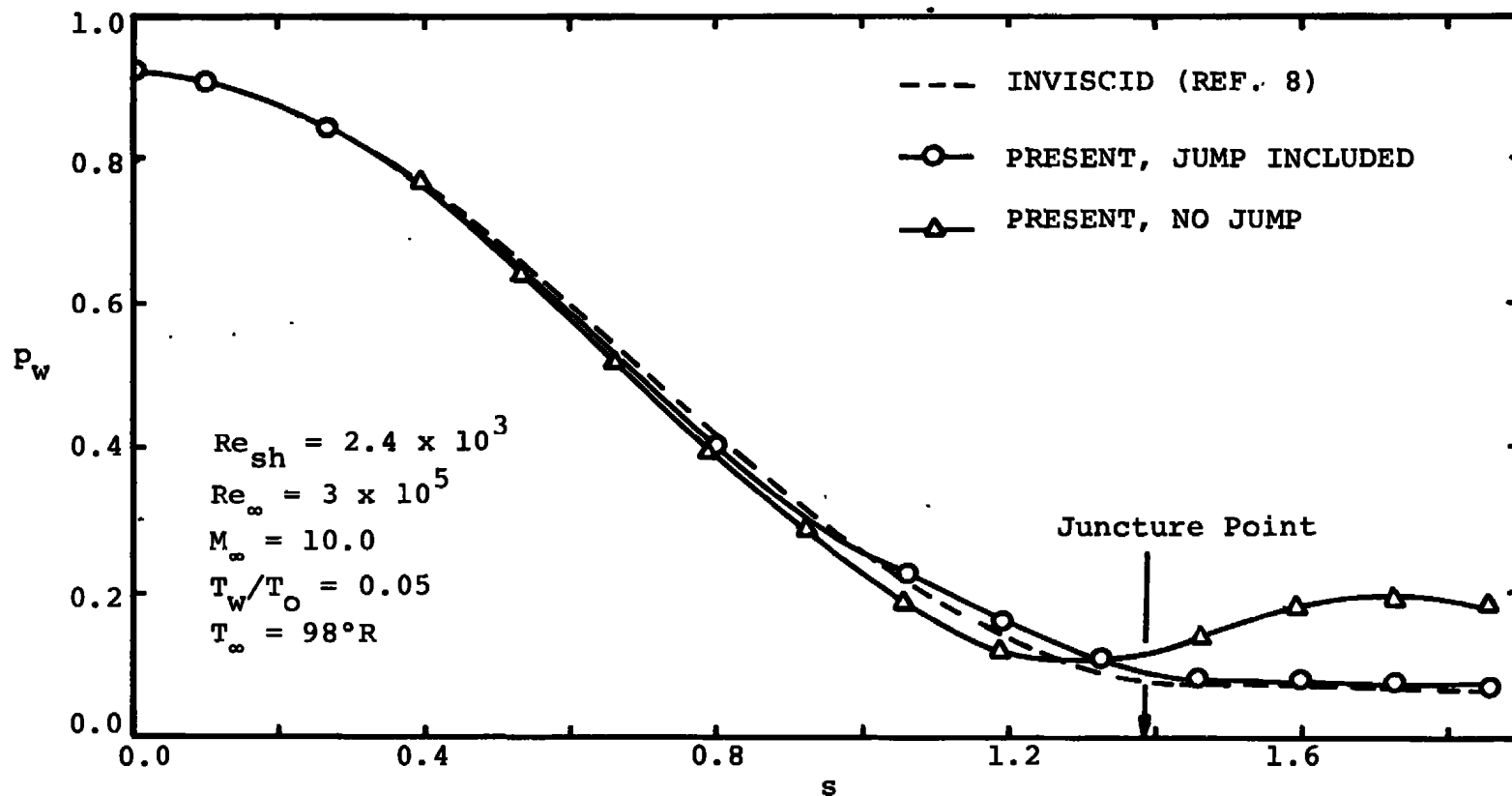
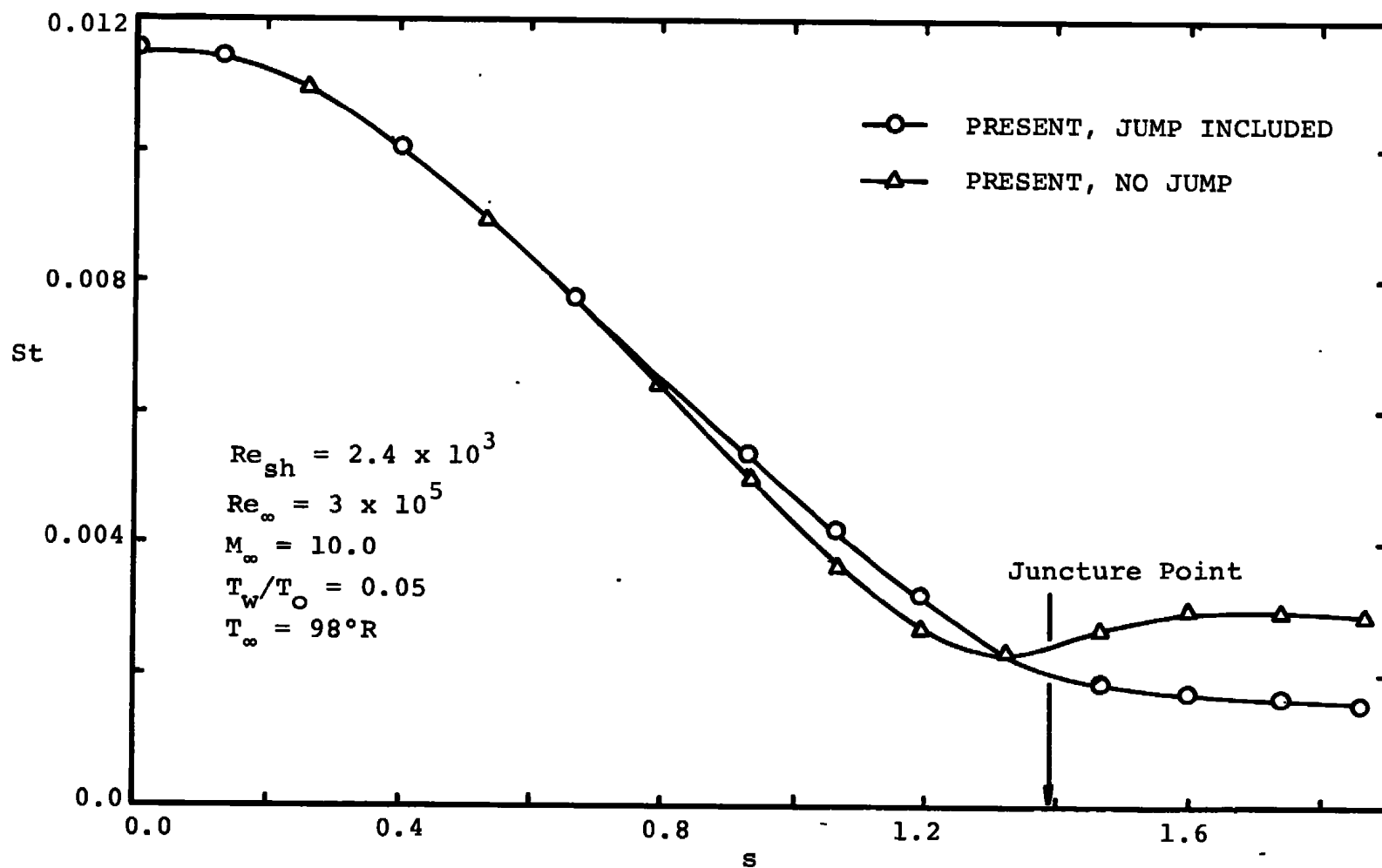
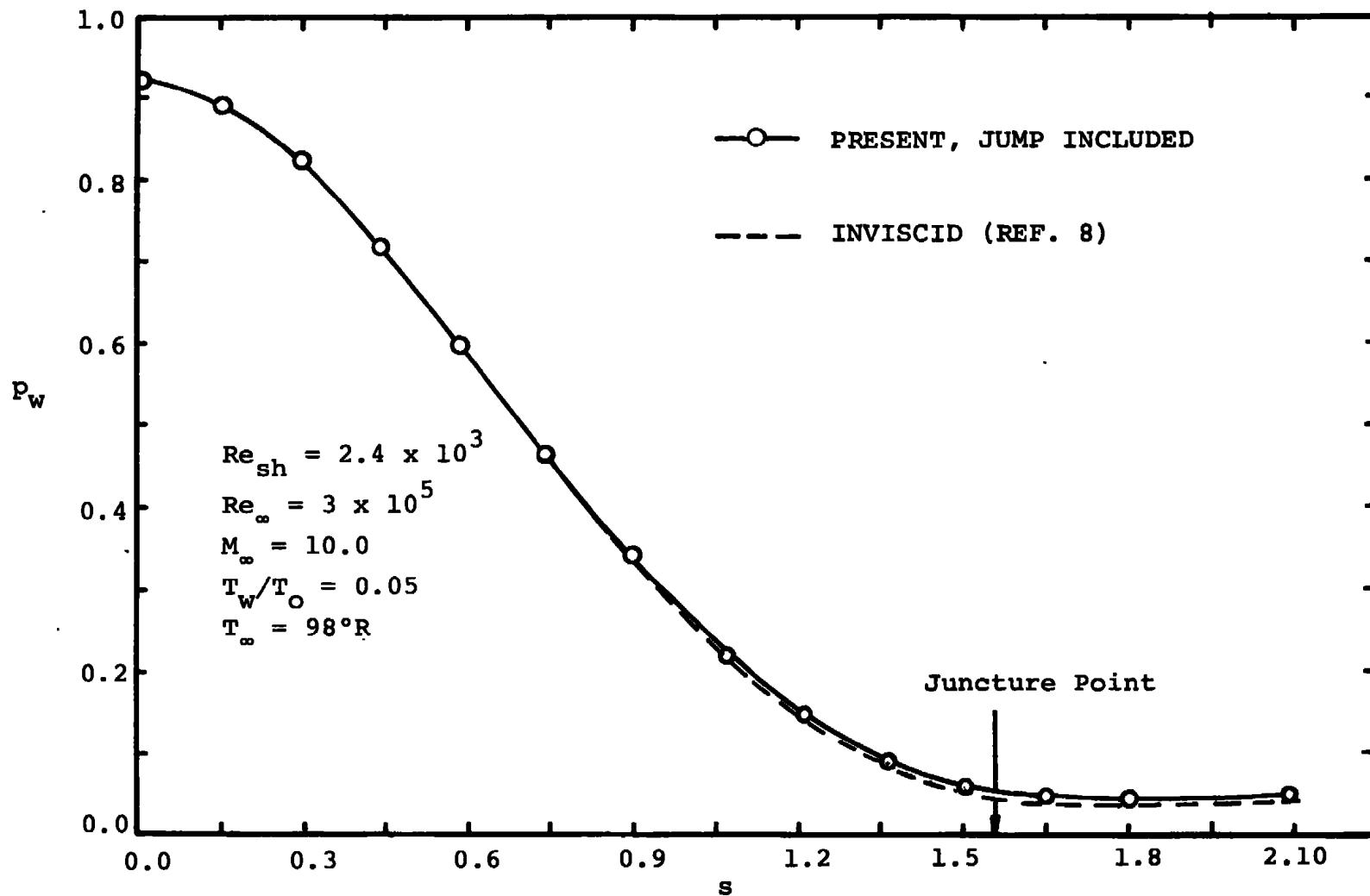


FIGURE 13 SURFACE PRESSURE DISTRIBUTION FOR A 10° SPHERE-CONE

FIGURE 14 HEAT TRANSFER DISTRIBUTION FOR A 10° SPHERE-CONE


 FIGURE 15 SURFACE PRESSURE DISTRIBUTION FOR A 0° SPHERE-CONE

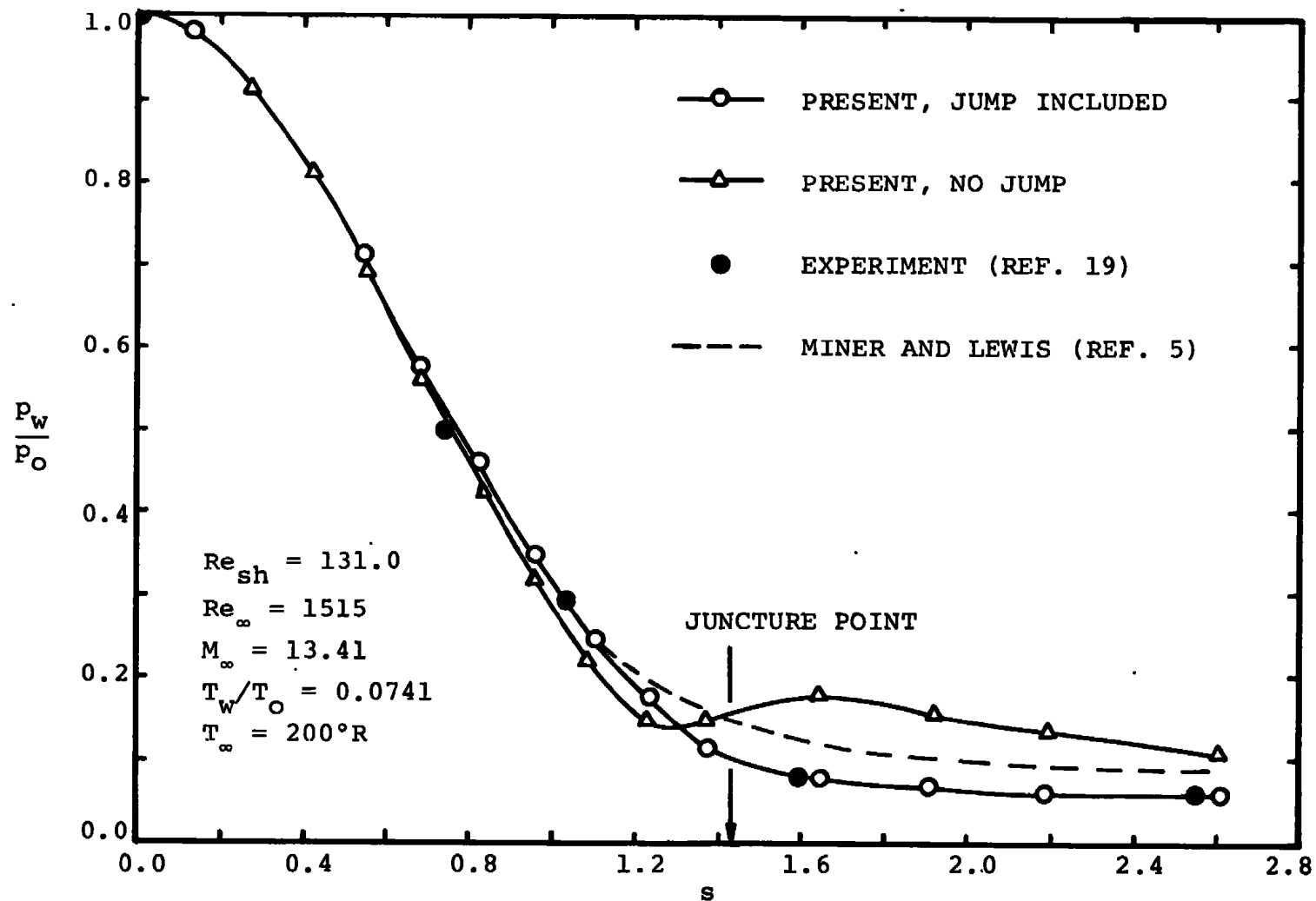
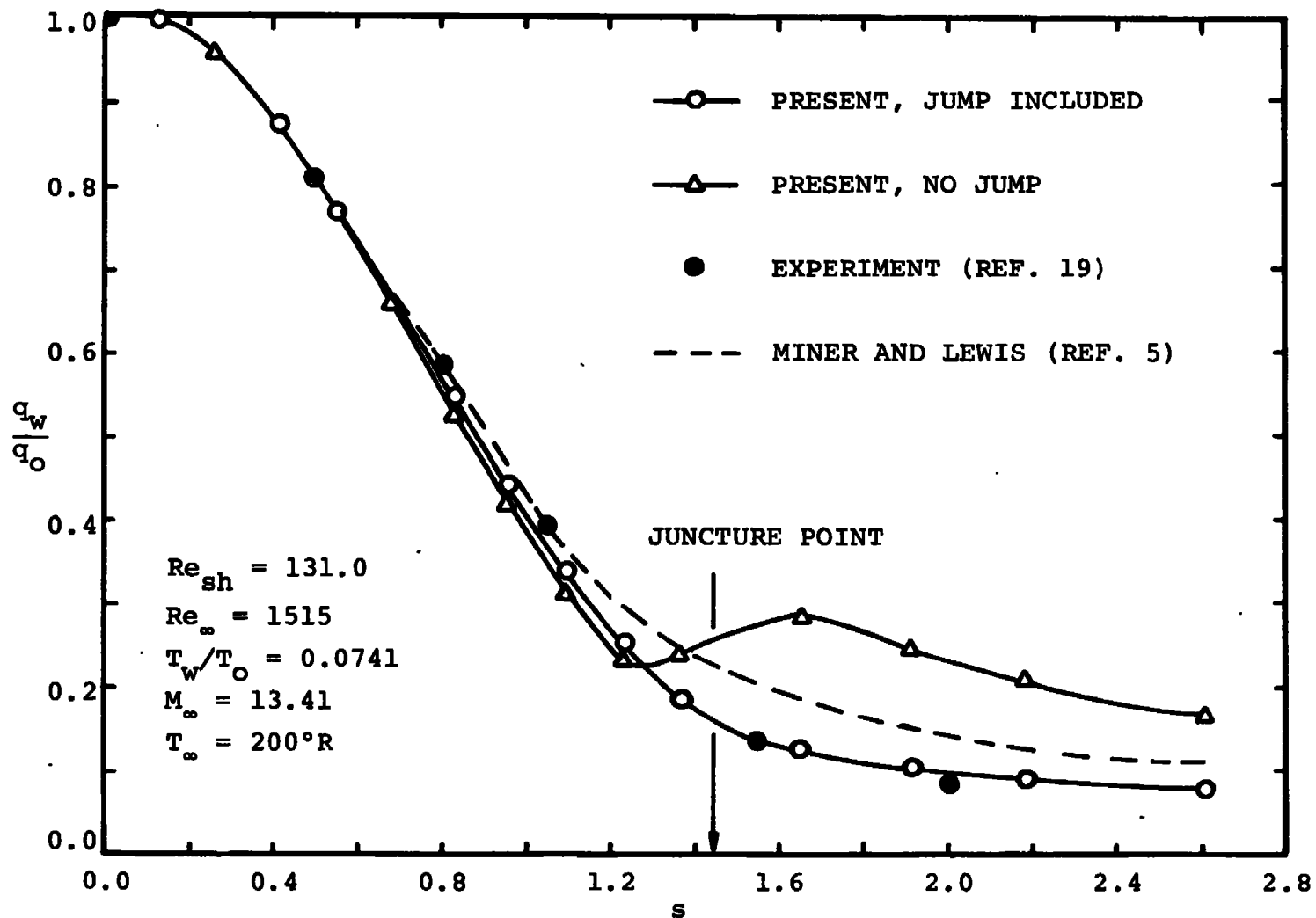


FIGURE 16 SURFACE PRESSURE DISTRIBUTION FOR A 7.5° SPHERE-CONE

FIGURE 17 HEAT TRANSFER DISTRIBUTION FOR A 7.5° SPHERE-CONE

APPENDIX A

ANALYSIS OF THE FLOW VARIABLES IN THE JUNCTURE REGION

The purpose of the present section is to study the analytical behavior of the flow field properties across a sphere/cone juncture point. In particular it is necessary to determine what, if any, restrictions the conservation laws place on the flow variables across a curvature discontinuity. For simplicity, only two dimensional flow is considered.

The analysis naturally begins with the integral form of the conservation laws since they alone are capable of accommodating discontinuities in the flow variables if they are called for. Since the viscous shock layer approach employs approximations to the differential form of the governing equations, it is first necessary to identify the equivalent approximations in the integral formulation of the problem. This is first performed for a fluid element located away from the juncture point with the results subsequently generalized to the juncture point.

To do this, first consider the infinitesimal element shown in the sketch below as being located at some point s, n away from the point of surface curvature discontinuity. Note that from the geometry,

$$\Delta s_3 = (1 + \kappa n - \kappa \Delta n) \Delta s \quad (A1)$$

$$\Delta s_4 = (1 + \kappa n + \kappa \Delta n) \Delta s \quad (A2)$$

and the unit vectors required in the conservation equations are given as

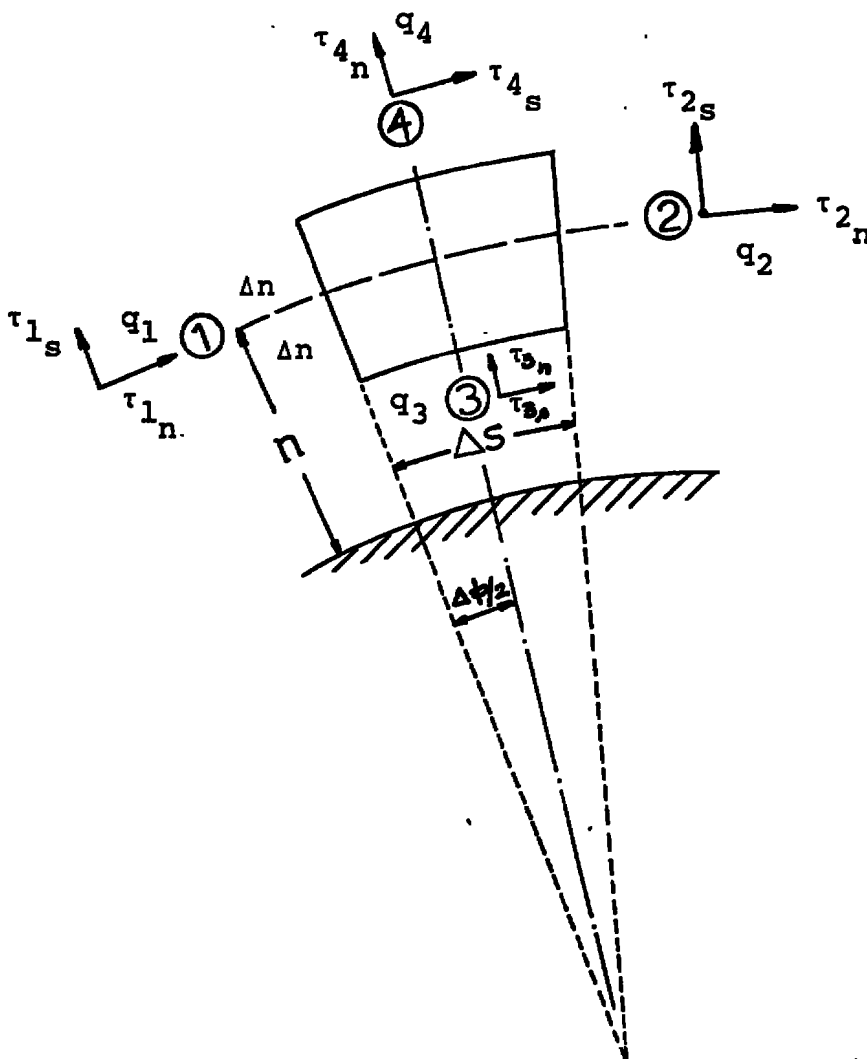
$$\vec{e}_{s_1} = \cos \frac{\Delta\phi}{2} \vec{e}_s + \sin \frac{\Delta\phi}{2} \vec{e}_n$$

$$\vec{e}_{s_2} = \cos \frac{\Delta\phi}{2} \vec{e}_s - \sin \frac{\Delta\phi}{2} \vec{e}_n$$

$$\vec{e}_{n_1} = \cos \frac{\Delta\phi}{2} \vec{e}_n - \sin \frac{\Delta\phi}{2} \vec{e}_s$$

$$\vec{e}_{n_2} = \cos \frac{\Delta\phi}{2} \vec{e}_n + \sin \frac{\Delta\phi}{2} \vec{e}_s$$

(A3)



For small, $\frac{\Delta\phi}{2}$, these yield,

$$\begin{aligned}
 \vec{e}_{s_1} &= \vec{e}_s + 1/2 \kappa \vec{e}_n \Delta s + \dots \\
 \vec{e}_{s_2} &= \vec{e}_s - 1/2 \kappa \vec{e}_n \Delta s + \dots \\
 \vec{e}_{n_1} &= \vec{e}_n - 1/2 \kappa \vec{e}_s \Delta s + \dots \\
 \vec{e}_{n_2} &= \vec{e}_n + 1/2 \kappa \vec{e}_s \Delta s + \dots
 \end{aligned} \tag{A4}$$

Also

$$\begin{aligned}
 \vec{e}_{s_3} &= \vec{e}_{s_4} = \vec{e}_s \\
 \vec{e}_{n_3} &= \vec{e}_{n_4} = \vec{e}_n
 \end{aligned}$$

The continuity equation in integral form is given as,

$$\iint (\rho \vec{v} \cdot \vec{n}) ds = 0 \tag{A5}$$

which can be evaluated on the element shown to yield,

$$(\rho_2 u_2 - \rho_1 u_1) 2\Delta n + [\rho_4 v_4 \Delta s_4 - \rho_3 v_3 \Delta s_3] = 0 \tag{A6a}$$

so that with application of equations (A1) and (A2) this gives

$$\begin{aligned}
 (\rho_2 u_2 - \rho_1 u_1) 2\Delta n + [\rho_4 v_4 (1 + \kappa n + \kappa \Delta n) \\
 - \rho_3 v_3 (1 + \kappa n - \kappa \Delta n)] \Delta s = 0
 \end{aligned} \tag{A6b}$$

so that as $\Delta s \rightarrow 0$

$$\rho_2 u_2 = \rho_1 u_1 \quad (\text{A7})$$

The differential form of this equation can be recovered using Taylor series expansion to write, for example, that

$$p_2 = p + \frac{\Delta s}{2} p_s + \dots \quad (\text{A8})$$

$$p_1 = p - \frac{\Delta s}{2} p_s + \dots \quad (\text{A9})$$

which, when used in equation (A6b) gives

$$(\rho u)_s + [(1 + \kappa n) \rho v]_n = 0 \quad (\text{A10})$$

The most important point to note here is that for this equation no terms are neglected in the thin shock layer approach and hence, the equivalent thin layer version of the mass conservation law across a general line is given by equation (A7). This is not the case for the momentum equation as shown below. The momentum equation in integral form is given as:

$$\iint [(\rho \vec{v} \cdot \vec{n}) \vec{v} + \rho \vec{n}] ds - \iint \vec{\tau} ds = 0 \quad (\text{A11})$$

Evaluating the first (inviscid) integral on the four sides of the control volume shown in the sketch yields

$$\begin{aligned} I_1 = & -[(\rho_1 u_1^2 + p_1) \vec{e}_{s_1} + \rho_1 u_1 v_1 \vec{e}_{n_1}] 2\Delta n + [(\rho_2 u_2^2 + p_2) \vec{e}_{s_2} \\ & + \rho_2 u_2 v_2 \vec{e}_{n_2}] 2\Delta n - [\rho_3 u_3 v_3 \vec{e}_{s_3} + (\rho_3 v_3^2 + p_3) \vec{e}_{n_3}] \Delta s_3 \\ & + [\rho_4 v_4 u_4 \vec{e}_{s_4} + (\rho_4 v_4^2 + p_4) \vec{e}_{n_4}] \Delta s_4 \end{aligned} \quad (\text{A12})$$

which is now rewritten using equations (A1)-(A4) to give

$$\begin{aligned}
 I_1 = & \{2\Delta n [(p_2 + \rho_2 u_2^2) - (p_1 + \rho_1 u_1^2) + 1/2 \kappa \Delta s (\rho_2 u_2 v_2 + \\
 & \cdot \rho_1 u_1 v_1)] + \Delta s [\rho_4 u_4 v_4 (1 + \kappa n + \kappa \Delta n) - \rho_3 u_3 v_3 (1 + \kappa n - \kappa \Delta n)]\} \vec{e}_s \\
 & + \{2\Delta n [\underline{\rho_2 u_2 v_2} - \underline{\rho_1 u_1 v_1} - 1/2 \kappa \Delta s (p_2 + \rho_2 u_2^2 + p_1 + \rho_1 u_1^2)] \\
 & + \Delta s [(p_4 + \underline{\rho_4 v_4^2}) (1 + \kappa n + \kappa \Delta n) - (p_3 + \underline{\rho_3 v_3^2}) (1 + \kappa n - \kappa \Delta n)]\} \vec{e}_n
 \end{aligned}$$

Similarly evaluating the viscous term yields,

$$\begin{aligned}
 I_2 = & (\tau_{1n} \vec{e}_{n1} + \tau_{1s} \vec{e}_{s1}) 2\Delta n + (\tau_{4s} \vec{e}_{s4} + \tau_{4n} \vec{e}_{n4}) \Delta s_4 \\
 & + (\tau_{2n} \vec{e}_{n2} + \tau_{2s} \vec{e}_{s2}) 2\Delta n + (\tau_{3n} \vec{e}_{n3} + \tau_{3s} \vec{e}_{s3}) \Delta s_4
 \end{aligned} \tag{A13}$$

In these relations those terms that would be dropped in the full shock layer approach have been identified with a single underscoring, while those additional terms whose contributions would be dropped in a thin layer approach have been given a double underscoring. These terms were identified here by first proceeding to the differential form of the governing equations, as was done for the continuity above, marking the shock layer approximations there and then

tracking these backward to their source terms in equations (A12) and (A13).

The most important result to emerge from these equations is that across a general line (i.e., $\Delta s \rightarrow 0$), the flow variables in a shock layer are governed by the conditions that

$$p_2 + \rho_2 u_2^2 = p_1 + \rho_1 u_1^2 \quad (\text{A14})$$

$$\underline{\rho_1 u_1 v_1} = \underline{\rho_2 u_2 v_2} \quad (\text{A15})$$

Combining the second of these with the continuity equation, (A7) requires that in a full layer approach the normal velocity be continuous across a line, while in the thin layer model, no such restriction is encountered.

The same procedure can now be applied to the integral form of the energy equation in viscous flows which is given as

$$\iint \rho \left(h + \frac{v^2}{2} \right) (\vec{v} \cdot \vec{n}) ds - \iint_S \vec{\tau} \cdot \vec{v} ds + \iint_S \vec{n} \cdot \vec{q} ds = 0 \quad (\text{A16})$$

Evaluating the first of these terms on the four sides of the element, as before, yields,

$$\begin{aligned} I_3 = & 2\Delta n [\rho_2 u_2 (h_2 + u_2^2/2 + v_2^2/2) - \rho_1 u_1 (h_1 + u_1^2/2 + v_1^2/2)] \\ & + \Delta s [\rho_4 v_4 (h_4 + u_4^2/2 + v_4^2/2) (1 + \kappa n + \kappa \Delta n) - \rho_3 v_3 \\ & (h_3 + u_3^2/2 + v_3^2/2) (1 + \kappa n - \kappa \Delta n)] \end{aligned} \quad (\text{A17})$$

Similarly the viscous terms become

$$\begin{aligned} \iint_S \vec{\tau} \cdot \vec{v} \, ds = & (\tau_{1n} u_1 + \tau_{1s} v_1) 2\Delta n + (\tau_{4s} u_4 + \tau_{4n} v_4) \Delta s_4 \\ & + (\tau_{2n} u_2 + \tau_{2s} v_2) 2\Delta n + (\tau_{3n} v_3 + \tau_{3s} u_3) \Delta s_3 \end{aligned} \quad (A18)$$

and

$$\iint_S \vec{n} \cdot \vec{q} \, ds = -\underline{q_1} 2\Delta n + q_4 \Delta s_4 + \underline{q_2} 2\Delta n - q_3 \Delta s_3 \quad (A19)$$

where again those terms equivalent to the shock layer approximation have been identified with a single underscoring and those terms equivalent to the present thin shock layer concept have been given a double underscoring.

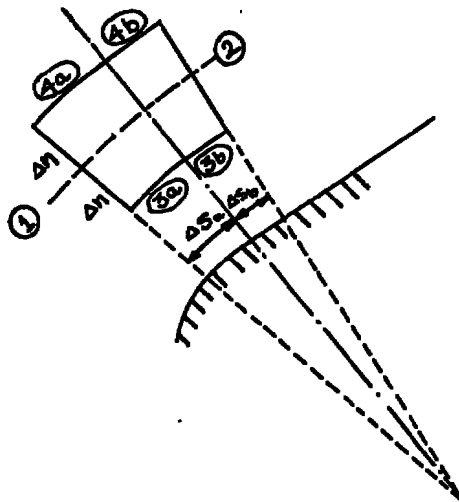
It is now possible to evaluate the resulting constraint on the flow variables across a line by setting $\Delta s \rightarrow 0$ to obtain

$$h_2 + u_2^2/2 + \underline{\underline{v_2^2/2}} = h_1 + u_1^2/2 + \underline{\underline{v_1^2/2}} \quad (A20)$$

Evaluation of this relation in combination with equation (A7), (A14), and (A15) verifies that whereas all variables ρ , u , v , and h must meet certain constraints across a line in the full shock layer approach, the present thin shock layer concept provides no constraint on the normal velocity v .

Now that the equivalent thin layer terms have been identified for a region of continuous surface curvature, one can proceed to the sphere/cone juncture point. For this study the element now straddles the juncture point and interest centers on the relation between the variables over faces 1 and 2 as Δs_a and $\Delta s_b \rightarrow 0$. With this in mind it is clear that the only terms of concern in the conservation laws are those integrals over the end faces 1 and 2. Thus, for example, only the first term of the continuity equation (A6a) need be considered and thus one can write immediately that across the juncture point

$$\rho_2 u_2 = \rho_1 u_1 \quad (A21)$$



For momentum conservation, equations (A12) and (A13) represent the appropriate point of departure except that now one cannot employ equations (A1)-(A2) and (A4). Nonetheless it is still clear that only those terms with Δn as a coefficient are of interest here and that all others can be ignored. With this in mind one can proceed to seek the limit form of the shock layer model as $\Delta s \rightarrow 0$ while keeping in mind the fact that on each side of the juncture line the shock layer approximations still hold. With this approach the results are identical to those presented in equations (A14) and (A15) for momentum and in equation (A20) for energy.

APPENDIX BDERIVATION OF SHOCK DERIVATIVES

The shock derivatives du_{sh}/ds , dT_{sh}/ds , dp_{sh}/ds and dv_{sh}/ds are derived in this appendix for use in the viscous shock layer solution.

In the spatial coordinate system the shock angle, α , is written as, (Figure 1)

$$\alpha = \tan^{-1} \left(\frac{dR}{dx_s} \right) \quad (B1)$$

$$\text{where } R = y_B + n_s \cos \phi \quad \text{and} \quad x_s = x_B - n_s \sin \phi \quad (B2)$$

Hence the derivative $d\alpha/ds$ is evaluated as,

$$\frac{d\alpha}{ds} = \frac{1}{[1 + (\frac{dR}{dx_s})^2]} \frac{d^2 R}{dx_s^2} \frac{dx_s}{ds} \quad (B3)$$

Note that

$$\frac{dx_s}{ds} = \cos \phi (1 + \kappa n_s) - n_s' \sin \phi \quad (B4)$$

and

$$\frac{dn_s}{ds} = (1 + \kappa n_s) \tan(\alpha - \phi) \quad (B5)$$

combining (B4) and (B5) yields

$$\frac{dx_s}{ds} = (1 + \kappa n_s) \frac{\cos \alpha}{\cos(\alpha - \phi)} \quad (B6)$$

substituting (B6) in (B3) and noting that $dR/dx_s = \tan\alpha$ yields after certain manipulations,

$$\frac{d\alpha}{ds} = (1+\kappa n_s) \frac{\cos^3 \alpha}{\cos(\alpha-\phi)} \frac{d^2 R}{dx_s^2} \quad (B7)$$

It is to be also noted that,

$$\tan\alpha = \frac{dR}{dx_s} = \frac{dR/ds}{dx_s/ds} \quad (B8)$$

Hence the second derivative, $d^2 R/dx_s^2$, can be shown to be

$$\frac{d^2 R}{dx_s^2} = \frac{d^2 R/ds^2}{(dx_s/ds)^2} - \frac{d^2 x_s/ds^2}{(dx_s/ds)^3} \frac{dR/ds}{dx_s/ds} \quad (B9)$$

Substituting for dx_s/ds from (B6) and then evaluating (B7) yields after proper manipulations,

$$\frac{d\alpha}{ds} = \frac{d^2 R}{ds^2} \left[\frac{\cos^2(\alpha-\phi)}{(1+\kappa n_s) \cos\phi} \right] - \frac{dR}{ds} \left[\frac{\kappa \sin(2\alpha-2\phi)}{\cos\phi(1+\kappa n_s)} \right] \quad (B10)$$

Note now that from Reference [16] equation (A-6) gives

$$\frac{du_{sh}}{ds} = K_1 \frac{d\alpha}{ds} - \kappa K_2 \quad (B11)$$

where

$$K_1 = \left(1 - \frac{\gamma-1}{\gamma} \frac{T_{sh}}{P_{sh}}\right) \cos(2\alpha+\beta) - \sin\alpha \cos(\alpha+\beta) \left(\frac{\gamma-1}{\gamma}\right) K_1' \quad (B12)$$

where K_1' is given by (A4) of Reference [16] and is of the form,

$$\frac{d}{ds} \left(\frac{T_{sh}}{P_{sh}} \right) = K_1' \frac{d\alpha}{ds} \quad (B13)$$

Also,

$$K_2 = \cos\alpha \cos(\alpha+\beta) + \frac{\gamma-1}{\gamma} \frac{T_{sh}}{P_{sh}} \sin\alpha \sin(\alpha+\beta) \quad (B14)$$

Likewise for other flow properties, [Ref. 16]

$$\frac{dp_{sh}}{ds} = K_3 \frac{d\alpha}{ds} \quad (B15)$$

where,

$$K_3 = \frac{2}{(\gamma+1)} \sin 2\alpha$$

$$\frac{dT_{sh}}{ds} = K_4 \frac{d\alpha}{ds} \quad (B16)$$

and

$$K_4 = \frac{2}{(\gamma+1)^2} \sin 2\alpha + \frac{4}{(\gamma+1)^2} \frac{\cos\alpha}{M_\infty^4 \sin^3\alpha}$$

$$\frac{dv_{sh}}{ds} = K_5 \frac{d\alpha}{ds} - K_6 \kappa \quad (B17)$$

where K_5 and K_6 are given by expression (A12) of Reference [16].

APPENDIX CCHARACTERISTICS OF THE SHOCK LAYER EQUATIONS

The characteristics are obtained from the inviscid full shock layer equations (17-22) and the corresponding "strip conditions" given by

$$du = \frac{\partial u}{\partial s} ds + \frac{\partial u}{\partial n} dn \quad (C1)$$

$$dv = \frac{\partial v}{\partial s} ds + \frac{\partial v}{\partial n} dn \quad (C2)$$

$$d\rho = \frac{\partial \rho}{\partial s} ds + \frac{\partial \rho}{\partial n} dn \quad (C3)$$

$$dp = \frac{\partial p}{\partial s} ds + \frac{\partial p}{\partial n} dn \quad (C4)$$

The terms in the normal momentum equation (19) which are not included in the thin shock layer version of the full shock layer equations will be marked here by introduction of a multiplicative factor, α , which would be zero for the thin shock layer equations.

Using Cramer's rule to identify the derivatives gives

$$\frac{\partial u}{\partial n} = |A|/|B| \quad (C5)$$

where

$$|B| = \begin{vmatrix} \frac{dn}{ds} & \frac{ds}{ds} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{dn}{ds} & \frac{ds}{ds} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{dn}{ds} & \frac{ds}{ds} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{dn}{ds} & \frac{ds}{ds} \\ 0 & \rho & (1+\kappa n)\rho & 0 & (1+\kappa n)v & u & 0 & 0 \\ (1+\kappa n)\rho v & \rho u & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & \alpha(1+\kappa n)\rho v & \alpha\rho u & 0 & 0 & (1+\kappa n) & 0 \\ 0 & 0 & 0 & 0 & -(1+\kappa n)va^2 & -ua^2 & (1+\kappa n)v & u \end{vmatrix}$$

$$|A| = \begin{vmatrix} \frac{du}{ds} & \frac{ds}{ds} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{dv}{ds} & 0 & \frac{dn}{ds} & \frac{ds}{ds} & 0 & 0 & 0 & 0 \\ \frac{d\rho}{ds} & 0 & 0 & 0 & \frac{dn}{ds} & \frac{ds}{ds} & 0 & 0 \\ \frac{d\rho}{ds} & 0 & 0 & 0 & 0 & 0 & \frac{dn}{ds} & \frac{ds}{ds} \\ -\kappa\rho v & \rho & (1+\kappa n)\rho & 0 & (1+\kappa n)v & u & 0 & 0 \\ -\kappa\rho uv & \rho u & 0 & 0 & 0 & 0 & 0 & 1 \\ \kappa\rho u^2 & 0 & \alpha(1+\kappa n)\rho v & \alpha\rho u & 0 & 0 & (1+\kappa n) & 0 \\ 0 & 0 & 0 & 0 & -(1+\kappa n)va^2 & -ua^2 & (1+\kappa n)v & u \end{vmatrix} \quad (C6)$$

Setting $|B| = 0$, expanding and simplifying yields

$$\begin{aligned} & [u \frac{dn}{ds} - (1+\kappa n)v \frac{ds}{ds}] \{ \alpha \frac{dn}{ds}^3 u (u^2 - a^2) - \alpha \frac{dn}{ds}^2 \frac{ds}{ds} u^2 (1+\kappa n)v \\ & - \alpha \frac{dn}{ds}^2 \frac{ds}{ds} v (1+\kappa n) (u^2 - a^2) - \frac{dn}{ds} \frac{ds}{ds}^2 u (1+\kappa n)^2 (a^2 - \alpha v^2) \\ & - \frac{ds}{ds} \frac{dn}{ds}^2 v (1+\kappa n) u^2 \alpha + \frac{ds}{ds}^2 \frac{dn}{ds} v^2 (1+\kappa n)^2 u \alpha \\ & + \alpha \rho^2 v^2 (1+\kappa n)^2 \frac{ds}{ds}^2 \frac{dn}{ds} u - (1+\kappa n)^3 \frac{ds}{ds}^3 v (-\alpha v^2 + a^2) \} = 0 \end{aligned} \quad (C7)$$

Note that

$$u \, dn - (1 + \kappa n) v \, ds = 0$$

gives the equation of a streamline in this coordinate system. With further simplification, the final expressions for the characteristics can be written as

$$\left(\frac{dn}{ds}\right)_{1,2} = \frac{\frac{-uv(1+\kappa n)}{a^2} \pm \frac{(1+\kappa n)}{a} \sqrt{\frac{1}{\alpha}(u^2 + \alpha v^2 - a^2)}}{(1 - u^2/a^2)} \quad (C8)$$

Note that when $\alpha=1$, which corresponds to the full shock layer equations, the characteristics are inclined at a Mach angle in the supersonic flow. However, as α approaches zero, corresponding to the thin shock layer version of these equations, the characteristics in the flow field tend to become perpendicular to the surface. This result can be verified by beginning with the thin layer equations and repeating the above derivation. For the thin layer case it can be shown that the characteristics are given as

$$- ds^2 \rho^2 a^2 (1 + \kappa n)^2 [dnu - (1 + \kappa n) v ds]^2 = 0 \quad (C9)$$

Note that this equation indicates that either

$$ds^2 = 0 \quad (C10)$$

$$\text{or} \quad [u \, dn - (1 + \kappa n) v \, ds]^2 = 0 \quad (C11)$$

Since (C11) represents the equation of a streamline in this coordinate system, the equation for the characteristics are given by (C10). Along these characteristics, compatibility conditions must be satisfied, these being obtained from,

$$ds^2 \rho (1+\kappa n) [-ua^2 dn + (1+\kappa n) va^2 ds] [-\rho \kappa u^2 dn + \rho (1+\kappa n) u du + (1+\kappa n) dp + \rho (1+\kappa n) \kappa uv ds] = 0 \quad (C12)$$

Note that this equation (C12) is satisfied along a characteristic line $ds=0$ indicating that no other additional condition need be satisfied. It is, therefore, seen that the inviscid set of thin shock layer equations predict coincident characteristics normal to the surface of the body. Davis [20] discusses the characteristics and the nature of these equations (i.e. whether they are elliptic, parabolic or hyperbolic) when viscous effects are included.

APPENDIX DADI FORMULATION OF S-MOMENTUM EQUATION

The s-momentum equation is written in the following form in the surface coordinate system.

$$\begin{aligned} \frac{\rho u u_s}{(1+\kappa n)} + \rho v u_n + \frac{\kappa u v \rho}{(1+\kappa n)} + \frac{p_s}{(1+\kappa n)} \\ = [\epsilon^2 / (1+\kappa n)^2 (r+n \cos \phi)^j] [(1+\kappa n)^2 (r+n \cos \phi)^j \tau]_n \end{aligned} \quad (D1)$$

where

$$\tau = \mu [u_n - \kappa u / (1+\kappa n)]$$

Using the transformation given in equations (5a-h) this becomes

$$\frac{\partial^2 \bar{u}}{\partial \eta^2} + \alpha_1 \frac{\partial \bar{u}}{\partial \eta} + \alpha_2 \bar{u} + \alpha_3 + \alpha_4 \frac{\partial \bar{u}}{\partial \xi} = 0 \quad (D2)$$

where α_1 , α_2 , α_3 and α_4 are given by equations (8a-d).

Note that u'_{sh} and p'_{sh} appear in the coefficients α_2 and α_3 . From Appendix (B) u'_{sh} and p'_{sh} are written as,

$$u'_{sh} = K_1 \frac{da}{ds} - K_2 \quad (D3)$$

$$p'_{sh} = K_3 \frac{da}{ds} \quad (D4)$$

where $d\alpha/ds$ is given by (B10) of Appendix (B). Substituting $d\alpha/ds$ in (D3) and (D4) and then evaluating the coefficients α_2 and α_3 yields

$$\alpha_2 = \gamma_2 \frac{d^2 R}{ds^2} + \gamma_3 \frac{dR}{ds} + \gamma_4 \quad (D5)$$

$$\alpha_3 = \gamma_5 \frac{d^2 R}{ds^2} + \gamma_6 \frac{dR}{ds} + \gamma_7 \quad (D6)$$

where,

$$\gamma_4 = +A\kappa K_2 + B + C + D \quad (D7)$$

$$\gamma_3 = +A K_1 \frac{K \sin(2\alpha - 2\phi)}{\cos\phi (1 + \kappa n_s)} \quad (D8)$$

$$\gamma_2 = -A K_1 \frac{\cos^2(\alpha - \phi)}{(1 + \kappa n_s) \cos\phi} \quad (D9)$$

and

$$A = + \frac{\rho_{sh} n_{sh}}{\epsilon^2 \mu_{sh}} \frac{\kappa n_{sh}}{1 + \kappa n_{sh} \eta} \frac{\bar{\rho} \bar{u}}{\bar{\mu}}$$

$$B = - \frac{\rho_{sh} v_{sh} n_{sh}}{\epsilon^2 \mu_{sh}} \frac{n_{sh}}{1 + \kappa n_{sh} \eta} \frac{\bar{\rho} \bar{v}}{\bar{\mu}}$$

$$C = - \frac{\kappa n_{sh}}{1 + \kappa n_{sh} \eta} \frac{\bar{\mu} n}{\bar{\mu}}$$

$$D = - \left(\frac{\kappa n_{sh}}{(1 + \kappa n_{sh} \eta)} + \frac{\cos\phi n_{sh}}{r + n_{sh} \cos\phi \eta} \right) \frac{\kappa n_{sh}}{(1 + \kappa n_{sh} \eta)}$$

In a similar manner,

$$\gamma_7 = A_1 \bar{p}_\xi - \frac{n_{sh}}{n_{sh}} \eta \bar{p}_\eta \quad (D10)$$

$$\gamma_6 = -A_1 \frac{\bar{p}}{p_{sh}} K_3 \frac{K \sin(2\alpha-2\phi)}{\cos\phi(1+\kappa n_{sh})} \quad (D11)$$

$$\gamma_5 = A_1 \frac{\bar{p}}{p_{sh}} \frac{\cos^2(\alpha-\phi)}{(1+\kappa n_{sh})\cos\phi} \quad (D12)$$

and

$$A_1 = - \frac{p_{sh} n_{sh}}{\epsilon^2 \mu_{sh}} \frac{n_{sh}}{(1+\kappa n_{sh} \eta)} \frac{1}{\bar{\mu} u_{sh}}$$

hence equation (D2) can now be written as

$$\begin{aligned} \frac{\partial^2 \bar{u}}{\partial \eta^2} + \alpha_1 \frac{\partial \bar{u}}{\partial \eta} + (\gamma_2 \frac{d^2 R}{ds^2} + \gamma_3 \frac{dR}{ds} + \gamma_4) \bar{u} \\ + (\gamma_5 \frac{d^2 R}{ds^2} + \gamma_6 \frac{dR}{ds} + \gamma_7) + \alpha_4 \frac{\partial \bar{u}}{\partial \xi} = 0 \end{aligned}$$

Further rearrangement yields,

$$\begin{aligned} \frac{\partial^2 \bar{u}}{\partial \eta^2} + \alpha_1 \frac{\partial \bar{u}}{\partial \eta} + (\gamma_2 \bar{u} + \gamma_5) \frac{d^2 R}{ds^2} + (\gamma_3 \bar{u} + \gamma_6) \frac{dR}{ds} \\ + (\gamma_4 \bar{u} + \gamma_7) + \alpha_4 \frac{\partial \bar{u}}{\partial \xi} = 0 \end{aligned} \quad (D13)$$

For the first half time step of the present alternating direction implicit method, this last expression is written as

First Sweep: $t^* = t^n + \Delta t/2$

$$\begin{aligned} \frac{\partial^2 \bar{u}^*}{\partial \eta^2} + \beta_1^* \frac{\partial \bar{u}}{\partial \eta} + \beta_2^* \left[\frac{\partial^2 R^n}{\partial s^2} - \frac{\partial R^*}{\partial t} \right] + \beta_3^* \frac{\partial R^n}{\partial s} \\ + \beta_4^* + \beta_5^* \frac{\partial \bar{u}^*}{\partial \xi} = 0 \end{aligned} \quad (D14)$$

For the second half time step equation (D13) is written as

Second Sweep: $t^{n+1} = t^* + \frac{\Delta t}{2}$

$$\begin{aligned} \beta_2^* \frac{\partial^2 R^{n+1}}{\partial s^2} - \beta_2^* \frac{\partial R^{n+1}}{\partial t} + \beta_3^* \frac{\partial R^{n+1}}{\partial s} + \left[\frac{\partial^2 \bar{u}}{\partial \eta^2} + \beta_1 \frac{\partial \bar{u}}{\partial \eta} \right. \\ \left. + \beta_5 \frac{\partial \bar{u}}{\partial \xi} + \beta_4 \right]^* = 0 \end{aligned} \quad (D15)$$

where,

$$\beta_1 = \alpha_1$$

$$\beta_2 = \gamma_2 \bar{u} + \gamma_5$$

$$\beta_3 = \gamma_3 \bar{u} + \gamma_6$$

$$\beta_4 = \gamma_4 \bar{u} + \gamma_7$$

$$\beta_5 = \alpha_4 \quad (D16)$$

However note that equation (D15) for R must be independent of η indicating that the coefficients of this equation must be independent of η .

It can be shown by proper substitution that,

$$\beta_3/\beta_2 = -2\kappa \tan(\alpha-\phi) \quad (D17)$$

whereas by using the first sweep equation

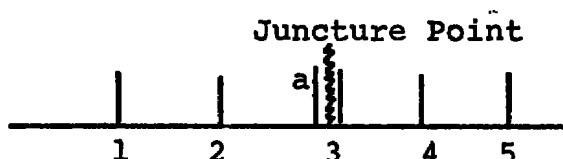
$$\begin{aligned} & [\frac{\partial^2 \bar{u}}{\partial \eta^2} + \beta_1 \frac{\partial \bar{u}}{\partial \eta} + \beta_5 \frac{\partial \bar{u}}{\partial \xi} + \beta_4] / \beta_2 \\ & = - \frac{\partial^2 R^n}{\partial s^2} + \frac{2R^*}{\Delta t} - \frac{2R^n}{\Delta t} + 2\kappa \tan(\alpha-\phi) \frac{\partial R^n}{\partial s} \end{aligned} \quad (D18)$$

Substituting all of this the final sweep equation is given as,

$$\begin{aligned} & \frac{\partial^2 R^{n+1}}{\partial s^2} - 2\kappa \tan(\alpha-\phi) \frac{\partial R^{n+1}}{\partial s} - \frac{2}{\Delta t} R^{n+1} - \frac{\partial^2 R^n}{\partial s^2} \\ & + 2\kappa \tan(\alpha-\phi) \frac{\partial R^n}{\partial s} + \frac{2}{\Delta t} (2R^* - R^n) = 0 \end{aligned} \quad (D19)$$

APPENDIX EDERIVATION OF FINITE DIFFERENCE EXPRESSIONSAT A JUNCTURE POINT

Consider a typical mesh system where a juncture point occurs immediately ahead of point 3. Point 'a' is taken to be immediately ahead of the juncture point.



Jump conditions associated with the first and second derivatives at the juncture point are given in the form

$$\left(\frac{dR}{ds}\right)_a = K_1 \left(\frac{dR}{ds}\right)_3 \quad (E1)$$

$$\left(\frac{d^2R}{ds^2}\right)_a = \left(\frac{d^2R}{ds^2}\right)_3 + K_2 \left(\frac{dR}{ds}\right)_3 + K_3 \quad (E2)$$

where K_1 , K_2 and K_3 are known and are given by equations (41b) for the model problem. Using Taylor's series expansion,

$$R_2 = R_a - \Delta R'_a + \frac{\Delta^2}{2} R''_a - \frac{\Delta^3}{6} R'''_a + \dots \quad (E3)$$

$$R_4 = R_3 + \Delta R'_3 + \frac{\Delta^2}{2} R''_3 + \frac{\Delta^3}{6} R'''_3 + \dots \quad (E4)$$

Rearranging this, (E3) can be rewritten using (E1) and (E2) as,

$$R_2 = R_3 - R_3' (K_1 - \frac{\Delta}{2} K_2) \Delta + \frac{\Delta^2}{2} K_3 + \frac{\Delta^2}{2} R_3'' - \frac{\Delta^3}{6} R_a''' + \dots \quad (E5)$$

so that finally,

$$R_3' = \frac{(R_4 - R_2)}{\Delta(K_1 - \frac{\Delta}{2} K_2 + 1)} + \frac{\Delta K_3}{2(K_1 - \frac{\Delta}{2} K_2 + 1)} + O(\Delta^2) \quad (E6)$$

Using (E4) and (E5) and simplifying yields,

$$R_3'' = \frac{(R_2 - R_3) + (K_1 - \frac{\Delta}{2} K_2)(R_4 - R_3)}{\frac{\Delta^2}{2} (K_1 - \frac{\Delta}{2} K_2 + 1)} - \frac{K_3}{\frac{\Delta^2}{2} (K_1 - \frac{\Delta}{2} K_2 + 1)} - \frac{\Delta}{3} \left[\frac{(K_1 - \frac{\Delta}{2} K_2) R_3''' - R_a'''}{(K_1 - \frac{\Delta}{2} K_2 + 1)} \right] + O(\Delta^2) \quad (E7)$$

Expression (E7) gives a formally first order accurate finite difference form of the second derivative immediately behind the juncture point. Note that the first order error in (E7) can be estimated for the simple model problem through differentiation of the differential equation on the two sides of the juncture point. From the model equation (38) differentiated and evaluating at "a" and "3", one obtains

$$R_a''' + \alpha_{1a} R_a'' + \alpha_2 R_a' = 0 \quad (E8)$$

$$R_3''' + \alpha_{13} R_3'' + \alpha_2 R_3' = 0 \quad (E9)$$

so that

$$\begin{aligned} R_a''' - (K_1 - \frac{\Delta}{2} K_2) R_3''' &= [\alpha_{13} (K_1 - \frac{\Delta}{2} K_2) - \alpha_{1a}] R_3'' \\ &- R_3' [K_2 \alpha_{1a} - \alpha_2 (K_1 - \frac{\Delta}{2} K_2) + \alpha_2 K_1] - K_3 \alpha_{1a} \end{aligned} \quad (E10)$$

The error term in (E7) can now be estimated as,

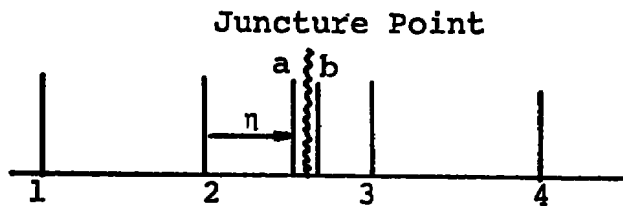
$$\begin{aligned} E = \frac{\Delta [\alpha_{13} (K_1 - \frac{\Delta}{2} K_2) - \alpha_{1a}]}{3(K_1 - \frac{\Delta}{2} K_2 + 1)} R_3'' - [R_3' \{ \alpha_{1a} K_2 - \alpha_2 (K_1 - \frac{\Delta}{2} K_2) \\ + \alpha_2 K_1 \} \Delta] + K_3 \alpha_{1a} \Delta \Big/ 3(K_1 - \frac{\Delta}{2} K_2 + 1) \end{aligned} \quad (E11)$$

Note that when (E11) is substituted in (E7) and formally the second order accurate finite difference form of the first derivative, dR/ds , is utilized, this would result in a formally second order accurate finite difference form of the second derivative, d^2R/ds^2 .

The above difference formulations are valid only when a mesh point of the finite difference scheme coincides with the juncture point where discontinuities in derivatives are encountered. These formulations need to be rederived

when the juncture point lies between two mesh points of the finite difference solution scheme. This is achieved in the following analysis.

Consider the mesh points as shown where the juncture point lies at a finite distance of away from point 2. Points a and b are immediately ahead and behind the juncture point.



Let η represent a fraction of the step size Δ . Thus,

$$\eta = \xi \Delta \quad 1 \leq \xi \leq 0 \quad (\text{E12})$$

Expressions are now sought for the shock derivatives at points "2" and "3" with an embedded jump occurring from "a" to "b". Using Taylor's series expansion,

$$R_1 = R_2 - \Delta R_2' + \frac{\Delta^2}{2} R_2'' - \frac{\Delta^3}{6} R_2''' + \dots \quad (\text{E13})$$

$$R_a = R_2 + \Delta \xi R_2' + \frac{\Delta^2}{2} \xi^2 R_2'' + \frac{\Delta^3}{6} \xi^3 R_2''' + \dots \quad (\text{E14})$$

$$R_3 = R_b + \Delta (1-\xi) R_b' + \frac{\Delta^2}{2} (1-\xi)^2 R_b'' + \frac{\Delta^3}{6} (1-\xi)^3 R_b''' + \dots \quad (\text{E15})$$

$$R_a' = R_2' + \Delta \xi R_2'' + \frac{\Delta^2 \xi^2}{2} R_2''' + \dots \quad (\text{E16})$$

$$R_a'' = R_2'' + \Delta \xi R_2''' + \dots \quad (\text{E17})$$

After considerable manipulations these give,

$$R_2' = \frac{(\xi+A)R_1 + R_3 + \frac{\Delta^2}{2}(1-\xi)^2 K_3 - R_2(\xi+A+1)}{\Delta^2 \left[\frac{\xi+A}{2} + \frac{\xi^2}{2} + \xi A + \frac{(1-\xi)^2}{2} \right]} + O(\Delta) \quad (E18)$$

and

$$R_2' = \{R_3 - 2R_1 P_2 - R_2(1-2P_2) + \frac{\Delta^2}{2}(1-\xi^2)K_3\} / [\Delta(P_1+P_2)] + O(\Delta^2) \quad (E19)$$

where

$$P_1 = \xi + A$$

$$P_2 = \frac{\xi^2}{2} + \xi A + (1-\xi)^2/2$$

In a similar fashion one obtains,

$$R_3' = [R_2 + R_4 P_1 - R_3(1+P_1) - \frac{\Delta^2}{2}\xi^2 K_3] / [(P_2+P_1/2)\Delta] + O(\Delta) \quad (E20)$$

and

$$R_3' = [2R_4 P_2 + R_3(1-2P_2) - R_2 + \frac{\Delta^2}{2}\xi^2 K_3] / [(P_1+2P_2)\Delta] + O(\Delta^2) \quad (E21)$$

where

$$P_1 = [\xi(K_1 - \frac{\Delta}{2}\xi K_2) + (1-\xi)]$$

$$P_2 = [\xi(1-\xi)(K_1 - \frac{\Delta}{2}\xi K_2) + \frac{\xi^2}{2} + \frac{1}{2}(1-\xi)^2]$$

APPENDIX FERROR ANALYSIS OF THE "SHOCK JUMP" MODEL PROBLEM

For the simple "shock jump" model problem studied in Section IV the exact solution is known and thus a detailed study of the accuracy of the finite difference scheme can be undertaken.

First, in order to establish the truncation error of the finite difference expression (44b) at the juncture point, the exact values of the shock shape, R , were used in this expression to obtain a numerical estimate to the second derivative, d^2R/ds^2 , at the point of discontinuity, i.e., at $S=0.9$. Figure 18 shows that this derivative linearly approaches its exact value in Δ , indicating that the finite difference form (44b) is indeed first order accurate at the point of discontinuity. Note also, from this figure that at any other point such as $S=0.5$ where no discontinuity of any kind is present the finite difference expression (44b) is seen to be second order accurate as anticipated since in such a case the jump constants K_1 , K_2 and K_3 take values of 1, 0 and 0 respectively. These results clearly indicate that the local truncation errors are of first order at a jump and second order everywhere else. It is, therefore, evident that the second order type behavior observed earlier can

only be explained through a study of the overall truncation error of the numerical scheme.

In order to better demonstrate this concept a simpler problem for which an analytical assessment of the error is possible, was considered. Thus we take the simple problem given as

$$\frac{d^2 R}{ds^2} - b^2 R = 0 \quad (F1)$$

Subject to the boundary conditions

$$R(0) = 0 \quad \text{and} \quad R(1) = 1 \quad (F2)$$

which has the exact solution as,

$$R = \left[\frac{e^{bs} - e^{-bs}}{e^b - e^{-b}} \right] \quad (F3)$$

Consideration is now given to three different schemes for numerically solving the same problem, the first two of which establish the method for assessing the overall truncation error and the last of which directly addresses the present problem.

Case (a) - Consider the case where $d^2 R/ds^2$ is represented by a second order accurate expression in the entire solution region. In such a case it is possible to show straightforwardly that the difference solution approaches the exact solution (F3) quadratically in Δs . Since this is the basis for later studies, it is shown below in detail.

The difference version of equation (F1) is given as

$$R_{i+1} - R_i(2 + b^2\Delta^2) + R_{i-1} = 0 \quad (F4)$$

$$R_0 = 0 \quad \text{and} \quad R_N = 1 \quad (F5)$$

which has a solution of the form

$$R_i = A s_1^i + B s_2^i \quad (F6)$$

where

$$s_{1,2} = 1 + \frac{b^2\Delta^2}{2} \pm \sqrt{\frac{b^4\Delta^4}{4} + b^2\Delta^2} \quad (F7)$$

Applying the boundary condition yields the final solution

$$R_i = \left[\frac{s_1^i - s_2^i}{s_1^N - s_2^N} \right] \quad (F8)$$

In the limit of zero step size, Δ , the difference solution (F8) should approach the exact solution (F3) quadratically in Δ . To verify this, consider now the limiting process, as $\Delta \rightarrow 0$. First write that

$$s_{1,2} = 1 \pm b\Delta + \frac{b^2\Delta^2}{2} \pm \frac{b^3\Delta^3}{8} + \dots \quad (F9)$$

and note that

$$s_1^i = s_1^{s/\Delta} \quad (F10)$$

which is now rewritten as

$$s_1^i = e^{s/\Delta \log_e(1+b\Delta)} e^{s/\Delta \log_e(1 + \frac{b^2 \Delta^2}{2})} \{1 - \frac{3b^3 \Delta^3}{8} - \frac{25}{128} b^5 \Delta^5 + \dots + \frac{3b^4 \Delta^4}{8} + \frac{25b^6 \Delta^6}{128} \dots\} s/\Delta$$

Expanding $\log_e(1+x)$ for small x and rewriting the resulting expression in terms of exponential functions we have,

$$s_1^i = e^{bs} e^{-b^2 \Delta s/2} e^{b^3 \Delta^2 s/2} \dots \times e^{b^2 \Delta s/2} e^{-b^4 \Delta^3 s/8} \times e^{b^6 \Delta^5 s/24} \dots e^{s/\Delta \log_e(1 - \frac{3b^3 \Delta^3}{8})} \{1 + \frac{3b^4 \Delta^4}{8} (1 - \frac{3b^3 \Delta^3}{8})^{-1} \dots\} s/\Delta$$

Note that the first order contribution in Δ , is precisely cancelled out yielding,

$$s_1^i = e^{bs} [(1 + \frac{5b^3 \Delta^2 s}{24} + \dots)(1 + \dots) \dots]$$

It is seen, therefore, that $s_1^{s/\Delta}$ approaches its exact solution, e^{bs} , quadratically in Δ .

Due to symmetry, a similar analysis for the remaining terms results in the following forms.

$$s_1^i = e^{bs} [1 + A\Delta^2 + B\Delta^3 + \dots]$$

$$s_2^i = \bar{e}^{bs} [1 + A_1\Delta^2 + B_1\Delta^3 + \dots]$$

$$s_1^N = e^b [1 + a\Delta^2 + b\Delta^3 + \dots]$$

$$s_2^N = \bar{e}^b [1 + a_1\Delta^2 + b_1\Delta^3 + \dots]$$

Hence the difference solution, as $\Delta \rightarrow 0$, can be written as

$$R_i = \frac{(e^{bs} - \bar{e}^{bs}) + \Delta^2 (Ae^{bs} - A_1\bar{e}^{bs}) + \Delta^3 (\dots)}{(e^b - \bar{e}^b) + \Delta^2 (ae^b - a_1\bar{e}^b) + \Delta^3 (\dots)} \quad (F11)$$

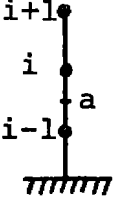
which can be manipulated to the form

$$R_i = \frac{e^{bs} - \bar{e}^{bs}}{e^b - \bar{e}^b} + O(\Delta^2)$$

Figure 19 shows the computational result verifying this analytical derivation where the function "R" and its derivative, d^2R/ds^2 , are seen to approach their exact values as a straight line in the square of the step size, Δ^2 .

Case (b) - Consider now a case when equation (F1) is solved with an algorithm that is first order accurate in the entire solution region. To do this the source term R of difference equation is written at the midpoint between two mesh points, point a, using the average value of R_i and R_{i-1} to approximate R_a . A centered second order

difference is used to represent R_i'' . The resulting first order accurate difference scheme is given as



$$R_{i+1}(2-b^2\Delta^2) - R_i(4+b^2\Delta^2) + 2R_{i-1} = 0 \quad (F12)$$

Using the same boundary conditions as before,

$$R_0 = 0, \quad R_N = 1 \quad (F13)$$

the difference equation solution is found to be

$$R_i = \frac{s_1^i - s_2^i}{s_1^N - s_2^N} \quad (F14)$$

where now

$$s_1 = 1 + b\Delta + \frac{3b^2\Delta^2}{4} + \frac{17b^3\Delta^3}{32} + \dots$$

$$s_2 = 1 - b\Delta + \frac{3b^2\Delta^2}{4} - \frac{17b^3\Delta^3}{32} + \dots$$

Following the same procedure of manipulation as before, it can be shown that

$$s_1^i = (e^{bs} e^{-b^2\Delta s/2} e^{b^3\Delta^2 s/3} \dots) \\ (e^{3/4 b^2\Delta s} e^{-9/16 b^4\Delta^3 s} \dots)$$

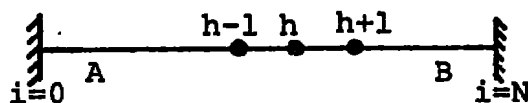
Note, here, that the first order term in Δ , does not cancel out as in the previous case and the final expression can be rewritten as

$$s_1^i = e^{bs} \left[\left(1 + \frac{b^2 \Delta s}{4} + \dots \right) (1 + \dots) \dots \right]$$

Figure 20 shows the computational result verifying that function "R" and its derivative d^2R/ds^2 approach their exact values linearly in Δ .

Case (c) - Consider now the case when one point in the finite difference scheme has a first order error while all other mesh points are formulated in a second order accurate sense. This case is then similar to the original model problem of Section IV where the local truncation error was second order at all points except one where a first order local error was encountered. Through study of the present problem one can more easily see how the introduction of a first order error at a single point does not cause the global truncation error to rise to a first order level.

For this study consider the following mesh system:



where in regions (A) and (B), the governing equation (F1), will be written using a second order accurate central difference expression while at the mesh point $i = h$, a first order accurate version of the governing equation (F1) will be used by again evaluating the source term R

between two mesh points. Note that this procedure is analogous to the earlier model problem (Eq. 38) where at the model juncture point the difference equation was first order accurate due to the jump effect while at all other points it was second order accurate. The advantage in the present simpler problem is that following the procedures of cases (a) and (b) a compact analytical error analysis can be made.

The difference equations to be solved are given as

$$R_{i+1} - R_i(2+b^2\Delta^2) + R_{i-1} = 0 \quad i = 1, \dots, h-1 \quad (F15)$$

$$R_{i+1}(2-b^2\Delta^2) - R_i(4+b^2\Delta^2) + 2R_{i-1} = 0 \quad i = h \quad (F16)$$

and

$$R_{i+1} - (2+b^2\Delta^2)R_i + R_{i-1} = 0 \quad i = h+1, \dots, N-1 \quad (F17)$$

subject to the boundary conditions

$$\begin{aligned} i = 0 \quad R_0 &= 0 \\ i = N \quad R_N &= 1 \end{aligned} \quad (F18)$$

A finite difference solution of (F15) between mesh points $i=1$ to $i=h-1$ yields the following result

$$R_i = R_h \left[\frac{s_1^i - s_2^i}{s_1^h - s_2^h} \right] \quad (F19)$$

whereas between mesh points $i=h+1$ to $i=N-1$ one obtains

$$R_i = A s_1^i + B s_2^i \quad (F20)$$

where,

$$A = \frac{R_h}{s_1^h} - \frac{s_2^h}{s_1^h} \left\{ \left(1 - \frac{R_h s_1^N}{s_1^h} \right) / \left(s_2^N - \frac{s_2^h s_1^N}{s_1^h} \right) \right\}$$

$$B = \left(1 - \frac{R_h s_1^N}{s_1^h} \right) / \left(s_2^N - \frac{s_2^h s_1^N}{s_1^h} \right)$$

and

$$s_1 = 1 + b\Delta + \frac{b^2 \Delta^2}{2} + \frac{b^3 \Delta^3}{8} + \dots$$

$$s_2 = 1 - b\Delta + \frac{b^2 \Delta^2}{2} - \frac{b^3 \Delta^3}{8} + \dots$$

Note that both solutions (F19) and (F20) depend on R_h which is still unknown. The difference equation (F16) for the "h" point relates R_h to R_{h+1} and R_{h-1} through the expression

$$R_h = \frac{R_{h+1} (2 - b^2 \Delta^2) + 2R_{h-1}}{(4 + b^2 \Delta^2)}$$

Since R_{h+1} and R_{h-1} are known in terms of R_h from equations (F19) and (F20), this equation would, after proper manipulation, yield an expression for R_h in terms of known quantities such as b and step size Δ . It is then possible to analytically assess the error term in this expression in the limit of zero step size. Note also, here, that this error would propagate to other mesh points through the solution (F19) ahead of this point and through the solution (F20) behind the mesh points. It is, therefore,

necessary to first estimate the error in " R_h " before an attempt is made to analyze the errors at any other points ahead or behind it. R_h can be rewritten in the following form

$$R_h = \frac{1}{2} \left[\frac{R_{h+1} \left(1 - \frac{b^2 \Delta^2}{2}\right) + R_{h-1}}{(1 + b^2 \Delta^4/4)} \right]$$

Substituting R_{h+1} from (F20) and R_{h-1} from (F19) and after some manipulations one obtains the following expression:

$$R_h \left[1 - \frac{b^2 \Delta^2}{2}\right] \frac{K_1}{K_2} \frac{s_1^h - s_2^h}{s_1^N - s_2^N} = \left(\frac{s_1^h - s_2^h}{s_1^N - s_2^N} \right) \left(1 - \frac{b^2 \Delta^2}{2}\right)$$

where,

$$K_1 = (s_2^N s_1^h - s_2^h s_1^N - s_2^N s_1^{h+1} + s_1^N s_2^{h+1})$$

and

$$K_2 = (s_1^h s_2^{h+1} - s_2^h s_1^{h+1})$$

Consider now the term

$$I = \frac{b^2 \Delta^2}{2} \frac{K_1}{K_2} \frac{s_1^h - s_2^h}{s_1^N - s_2^N}$$

which can be reformulated as,

$$I = \frac{b^2 \Delta^2}{2} \left[\frac{s_2^N}{s_2^h} \left(\frac{1-s_1}{s_2-s_1} \right) - \frac{s_1^N}{s_1^h} \left(\frac{1-s_2}{s_2-s_1} \right) \right] \left(\frac{s_1^h - s_2^h}{s_1^N - s_2^N} \right)$$

Noting that,

$$\frac{1 - s_1}{s_2 - s_1} = +\frac{1}{2} + \frac{b\Delta}{4} - \frac{b^3\Delta^3}{32}$$

and

$$\frac{1 - s_2}{s_2 - s_1} = -\frac{1}{2} + \frac{b\Delta}{4} + \frac{b^3\Delta^3}{32} \dots$$

$$\begin{aligned} R_h = & \left(\frac{s_1^h - s_2^h}{s_1^N - s_2^N} \right) \left[1 - \frac{b^2\Delta^2}{2} \right] \left[1 + \frac{b^2\Delta^2}{2} \left\{ \frac{s_2^N}{s_2^h} \left(\frac{1}{2} + \frac{b\Delta}{4} \right. \right. \right. \\ & \left. \left. \left. - \frac{b^3\Delta^3}{32} \dots \right) - \frac{s_1^N}{s_1^h} \left(-\frac{1}{2} + \frac{b\Delta}{4} + \frac{b^3\Delta^3}{32} + \dots \right) \right\} \left(\frac{s_1^h - s_2^h}{s_1^N - s_2^N} \right) \right. \\ & \left. + \dots \right] \end{aligned}$$

Expanding this,

$$\begin{aligned} R_h = & \frac{s_1^h - s_2^h}{s_1^N - s_2^N} + \frac{s_1^h - s_2^h}{s_1^N - s_2^N} \frac{b^2\Delta^2}{2} \left\{ \frac{s_2^N}{s_2^h} \left(\frac{1}{2} + \frac{b\Delta}{4} - \frac{b^3\Delta^3}{32} \dots \right) \right. \\ & \left. - \frac{s_1^N}{s_1^h} \left(-\frac{1}{2} + \frac{b\Delta}{4} + \frac{b^3\Delta^3}{32} + \dots \right) \right\} \left(\frac{s_1^h - s_2^h}{s_1^N - s_2^N} \right) \\ & - \frac{s_1^h - s_2^h}{s_1^N - s_2^N} \frac{b^2\Delta^2}{2} - \frac{b^4\Delta^4}{4} \{ \dots \} + \dots \end{aligned}$$

The extraction of the error term from this expression as Δ approaches zero is a process involving further manipulations, however if it is first noted that

$$s_1^h = e^{bs_h} [1 + A \Delta^2 + B \Delta^3 + \dots]$$

Where A and B are known constants, and that a similar statement holds for the terms s_2^h , s_1^N and s_2^N , then it is possible to rewrite R_h as

$$R_h = \left[\frac{e^{bs_h} - e^{-bs_h}}{e^b - e^{-b}} \right] + O(\Delta^2)$$

It is, thus, observed that the first order local truncation error at point "h" contributes to higher order global errors and the difference solution still approaches the exact solution quadratically in Δ . Note also that one can assess the error in R'' from the differential equation

$$\left(\frac{d^2 R}{ds^2} \right)_h = (b^2 R)_h$$

so that on substituting for R_h , this finite difference solution gives

$$\left(\frac{d^2 R}{ds^2} \right)_h = b^2 \left[\frac{e^{bs_h} - e^{-bs_h}}{e^b - e^{-b}} \right] + b^2 O(\Delta^2)$$

Hence

$$\left(\frac{d^2 R}{ds^2} \right)_h = \left(\frac{d^2 R}{ds^2} \right)_{\text{exact}} + b^2 O(\Delta^2)$$

indicating that the finite difference second derivative solution would also approach the exact solution quadratically in Δ . It is now possible to direct attention to the analytical difference solutions in regions (A) and (B), ahead and behind of the present mesh point "h". From equation (F19) it is directly seen in the light of previous derivation and the fact that R_h has a second order error associated with it, that the difference solution in region (A) would approach the exact solution as second order accurate in the limit of zero step size, Δ . The same result is also true for the region (B), since this region is written as second order accurate. Figure F-2 shows the computational result for this case where a first order accurate difference equation was used at a point $s = 0.5$ while all other mesh points were written in a second order accurate sense. The function R and its derivative d^2R/ds^2 are seen to approach their exact value as a straight line in the square of the step size, Δ , verifying the present analytical result. There remains now only the question as to when would this first order local truncation error in the numerical scheme produce an explicit first order global error. This point is addressed through Figures F-3 and F-4. It is seen from Figure F-4 that when a computational scheme utilizes the first order difference equation in a fixed

region of the entire solution regime, i.e. between $0.3 \leq s \leq 0.7$, the function "R" and its derivative, d^2R/ds^2 approach the exact solution at a point $s=0.5$ linearly in Δ indicating that the overall truncation error is of first order. Figure F-3 shows a similar study where the first order difference equation was only used on a fixed number of points, i.e. $(N)_{0.5} - 3 \leq (N)_{0.5} \leq (N)_{0.5} + 3$, in the entire solution regime. This figure shows that in this case the function "R" and its derivative, d^2R/ds^2 approach the exact solution as second order accurate scheme in the step size indicating that in this case the overall error is of second order. It is, thus, clear that a local error of order Δ will not sum up to a global error of order Δ if it only occurs at a finite number of points as the mesh is refined. The global error will only rise up to first order level if an infinite number of points contribute a first order local error as the mesh is refined.

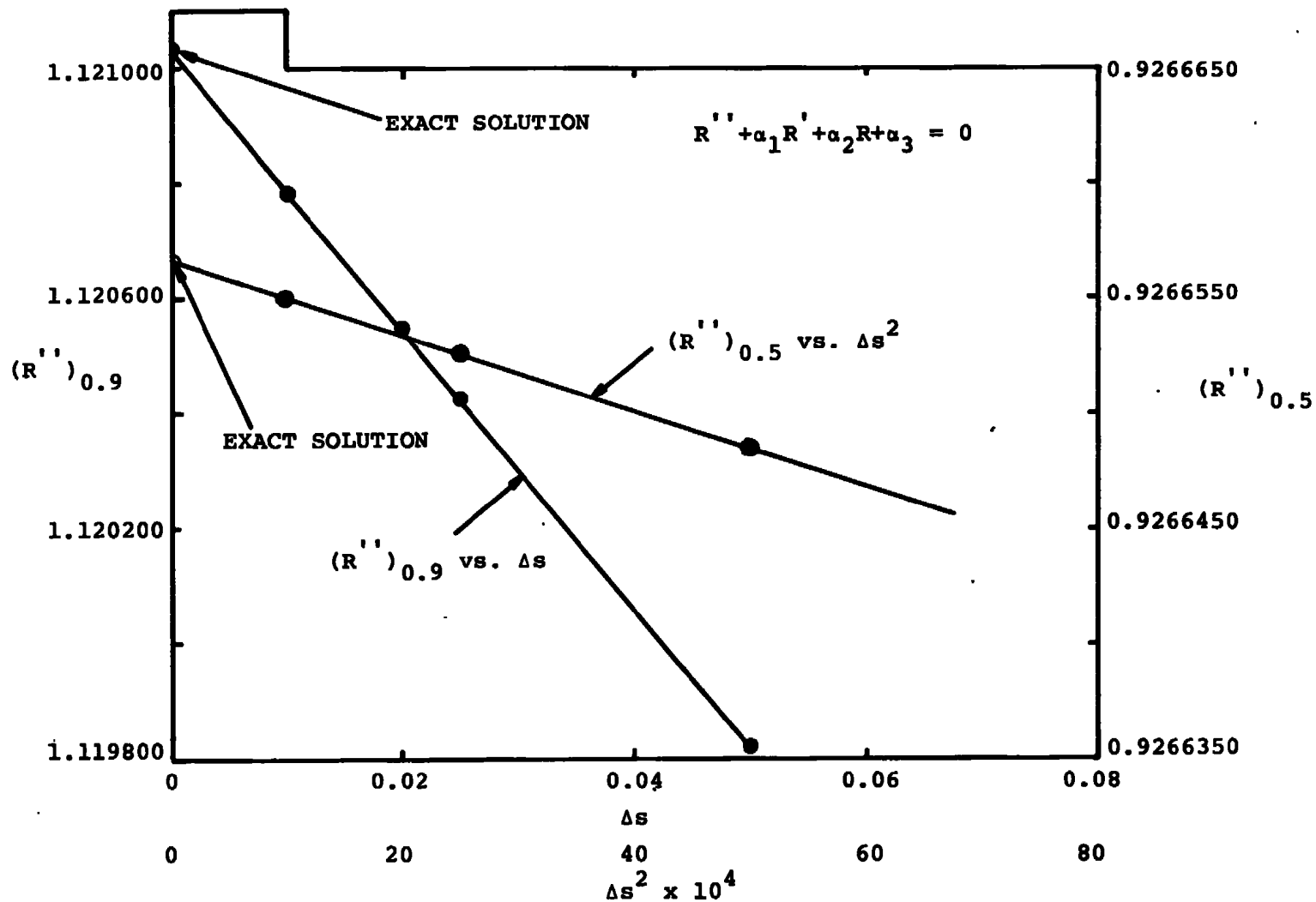


FIGURE F-1 MODEL STEP SIZE STUDY

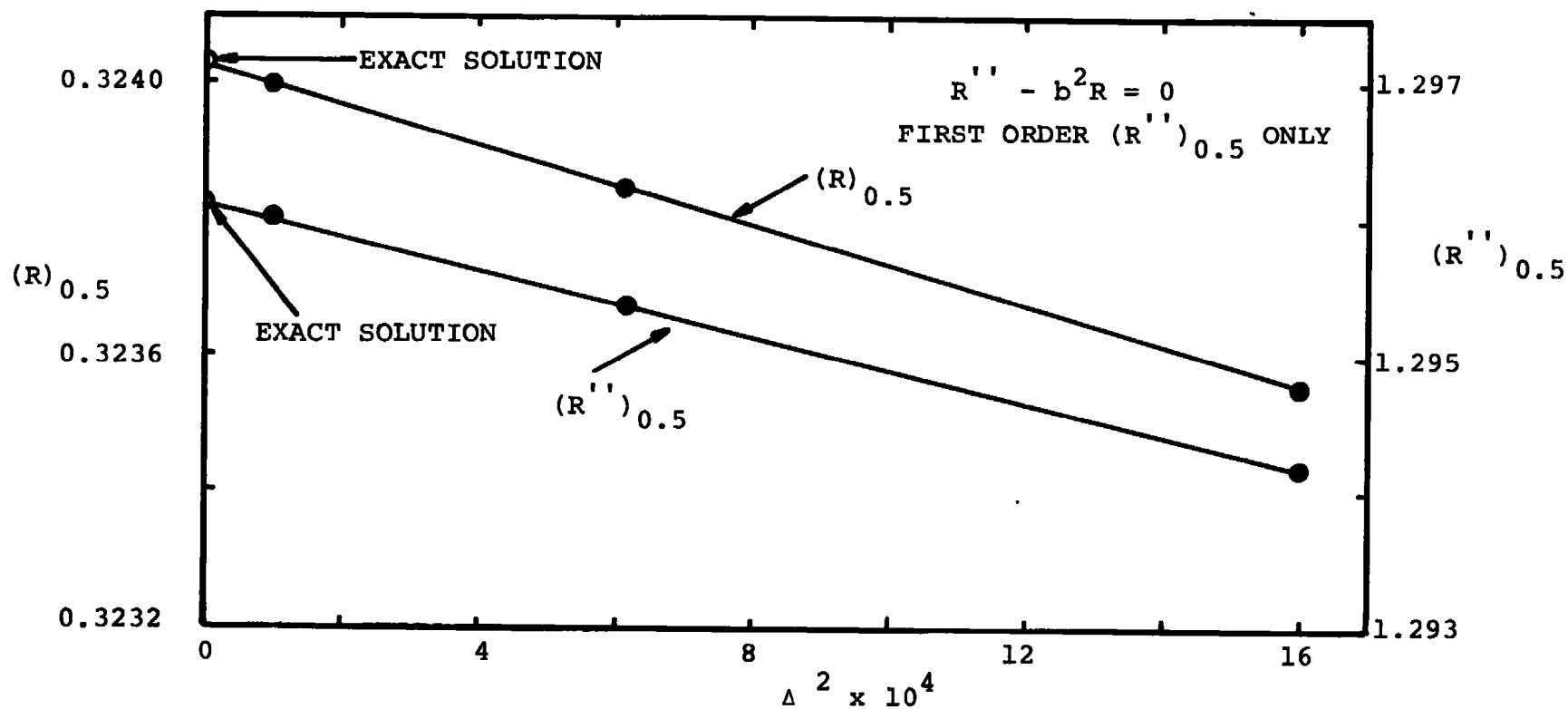


FIGURE F-2 MODEL PROBLEM ERROR STUDY

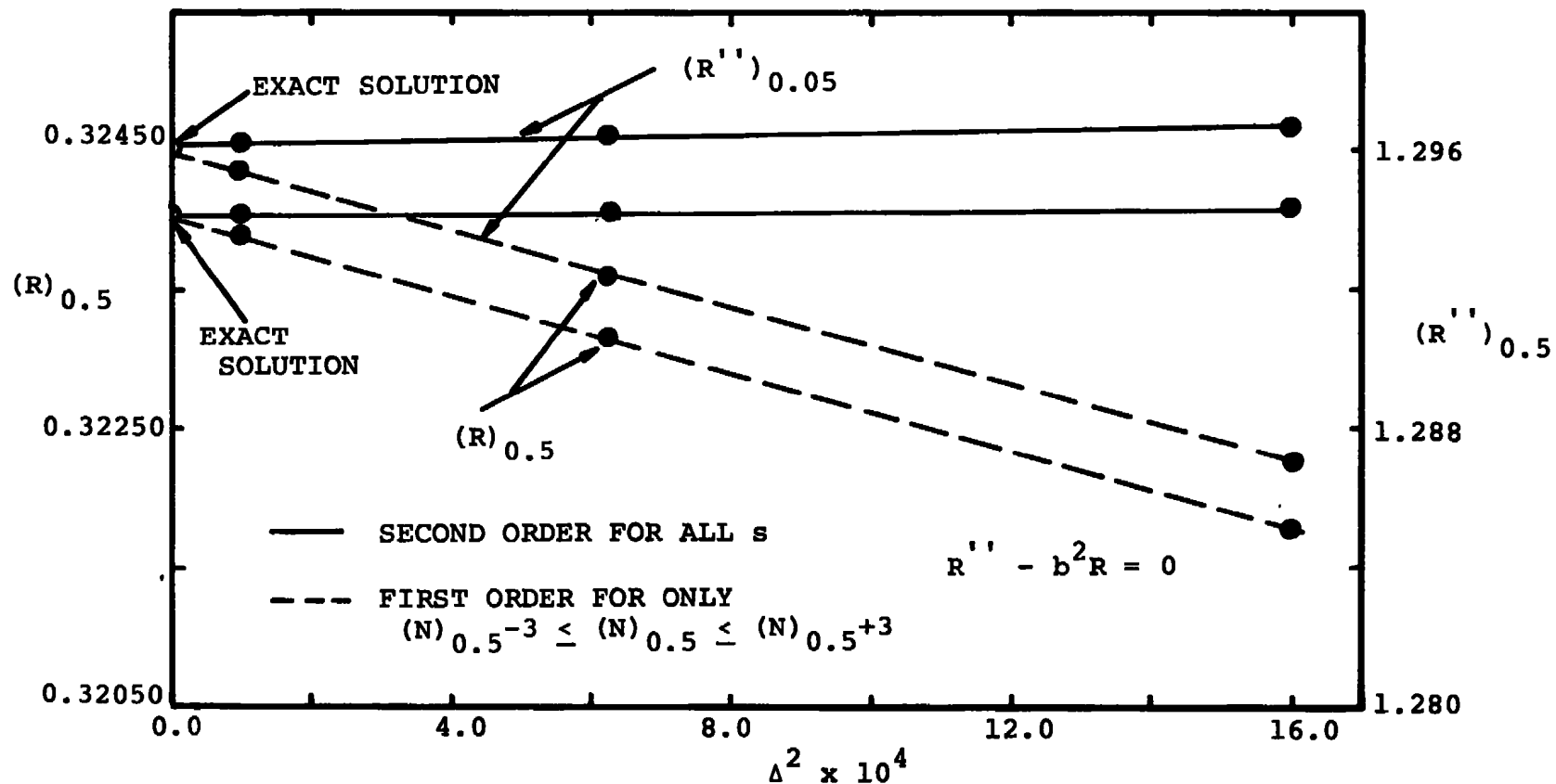


FIGURE F-3 MODEL PROBLEM ERROR STUDY

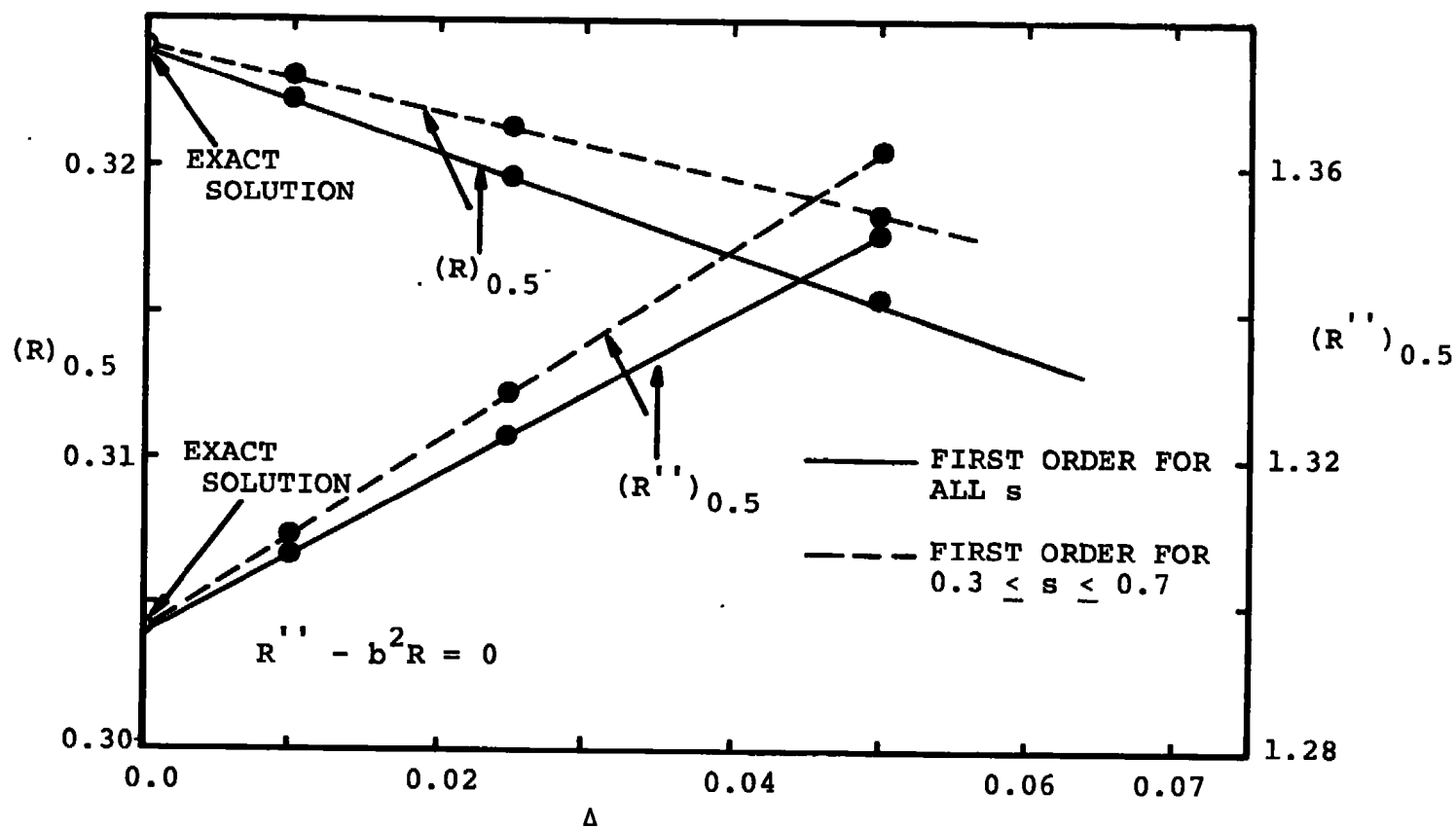


FIGURE F-4 MODEL PROBLEM ERROR STUDY

APPENDIX GNUMERICAL EVALUATION OF THE JUNCTURE POINT JUMP CONDITIONS

The first and second time sweep of the numerical scheme for the viscous shock layer solution has been discussed in Appendix (D) as represented by equations (D14) and (D15). Both of the time sweep solutions require information about the jumps associated with the terms, $\partial^2 R / \partial s^2$ and $\partial R / \partial s$ at the juncture point. The jump condition associated with the first derivative, dR/ds , is obtained straightforwardly from geometric considerations as

$$\left(\frac{dR}{ds}\right)_{\text{sphere}} = (1+n_s) \left(\frac{dR}{ds}\right)_{\text{cone}} \quad (G1)$$

However the jump condition associated with, $d^2 R / ds^2$ must be obtained from the momentum equation (D13), which may be rewritten in the form

$$\begin{aligned} \frac{d^2 R}{ds^2} - 2\kappa \tan(\alpha - \phi) \frac{dR}{ds} + \left[\frac{\bar{u}_{nn} + \alpha_1 \bar{u}_n + \gamma_4 \bar{u} + \gamma_7 + \alpha_4 \bar{u}_\xi}{(\gamma_2 \bar{u} + \gamma_5)} \right] \\ = 0 \end{aligned} \quad (G2)$$

Note that the last term in (G2) must be independent of η . Hence, evaluating (G2) on the two sides of the juncture point yields,

$$(R'')_{\text{sphere}} - 2 \tan(\alpha - \phi) (R')_{\text{sphere}} + (Fe)_{\text{sphere}} = 0 \quad (\text{G3})$$

$$(R'')_{\text{cone}} + (Fe)_{\text{cone}} = 0 \quad (\text{G4})$$

where,

$$Fe = \left[\frac{\bar{u}_{\eta\eta} + \alpha_1 \bar{u}_{\eta} + \gamma_4 \bar{u} + \gamma_7 + \alpha_4 \bar{u}_{\xi}}{(\gamma_2 \bar{u} + \gamma_5)} \right] \quad (\text{G5})$$

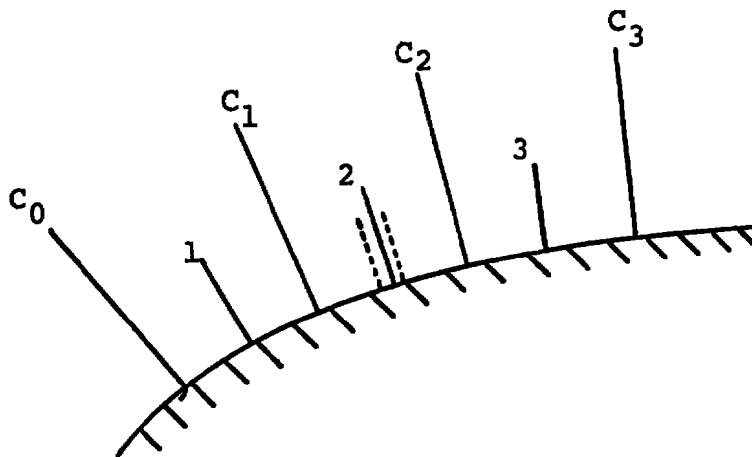
These two can be combined to give

$$\begin{aligned} (R'')_{\text{sphere}} &= (R'')_{\text{cone}} + 2 \tan(\alpha - \phi) (R')_{\text{sphere}} \\ &+ (Fe)_{\text{cone}} - (Fe)_{\text{sphere}} = 0 \end{aligned} \quad (\text{G6})$$

The term "Fe" should be a constant across the shock layer since the associated equation (G2) is independent of the normal coordinate, η . This was verified numerically at every stage of the calculation procedure for the viscous shock layer code. It was also found that there usually was a point in the shock layer where the denominator $(\gamma_2 \bar{u} + \gamma_5)$ in equation (G6) would pass through zero. It is, therefore, obvious that at such a point the term "Fe" would be in error due to numerical truncation process and care should be exercised to avoid any such region while evaluating these jump conditions. For the present viscous

shock layer code the jump conditions were evaluated at the first grid point away from the wall.

Another difficulty that was encountered in the numerical evaluation of the shock jump conditions stemmed from the manner in which the s-momentum equation (D2, Appendix D), was solved in the present form of the viscous shock layer code. The sketch below shows a typical finite difference mesh configuration for the present scheme.



Due to the nature of the ADI algorithm as applied here, it was necessary to solve the star time sweep equations for the flow properties \bar{u} , etc. at the numbered points "1", "2", "3" etc., while the final time sweep equations were solved for the shock shape at points C_0 , C_1 , C_2 , C_3 etc. The jump conditions given by equations (G1) and (G6) were to be applied in the middle of the second sweep

mesh from points "a" to "b" of the sketch. This then requires that the driving function, F_e , defined in equation (G5) be evaluated at points "a" and "b". To do this properly, the value of F_e at "a" was obtained by extrapolating its values at C_0 and C_1 to "a" while the value at "b" was obtained through extrapolation of F_e 's values at points C_2 and C_3 . The results thus achieved were found to be consistent throughout the calculations.

APPENDIX HCOMPUTER CODE FOR THE FULL VISCOUS SHOCK LAYEREQUATIONS FOR SPHERICALLY BLUNTED CONES

The following computer code, written in Fortran IV was used to obtain numerical solution of the full viscous shock layer equations for hypersonic flow past spherically blunted cones. The input quantities are:

Main Program

DT	Time step size.
IE	Number of mesh points in the η -direction.
IEND	Number of mesh points in the s -direction.
REYIN	Free stream Reynolds number, Re_∞ .
RMAC	Free stream Mach number, M_∞ .
BO	Wall to stagnation temperature ratio, T_w/T_o .
TEMP	Free stream temperature, T_∞ in degree Rankine.
GAM	Ratio of specific heats, γ .
SIGM	Prandtl number, σ .
XFACT	Convergence criterion for solving the governing equations by iteration.
THETA1	Sphere/cone angle for which solution is desired, θ .
THETA	Sphere/cone angle whose solution is used as an initial guess on the shock shape.

DTHETA Increment parameter which controls the increment in angle, $\Delta\theta$.

W Under relaxation parameter required during a profile iteration procedure.

WW Relaxation parameter used for two consecutive final sweeps.

J η -point across the shock layer, where shock jump conditions are evaluated, also convergence criteria put for profile iteration.

NJNC Number of mesh points between juncture and stagnation point.

DY Normal step size, $\Delta\eta$.

NITER Number of profile iterations.

NTIME Number of time cycles.

Input Parameters

THIN Positive when thin shock layer equations used.

THINI Negative when full shock layer equations used.

RUMP Positive when jump conditions are included.

SWFAC Positive when wall slip included.

SSFAC Positive when shock slip included.

AHALF Positive when an initial guess for half the longitudinal step size of input guess needed.

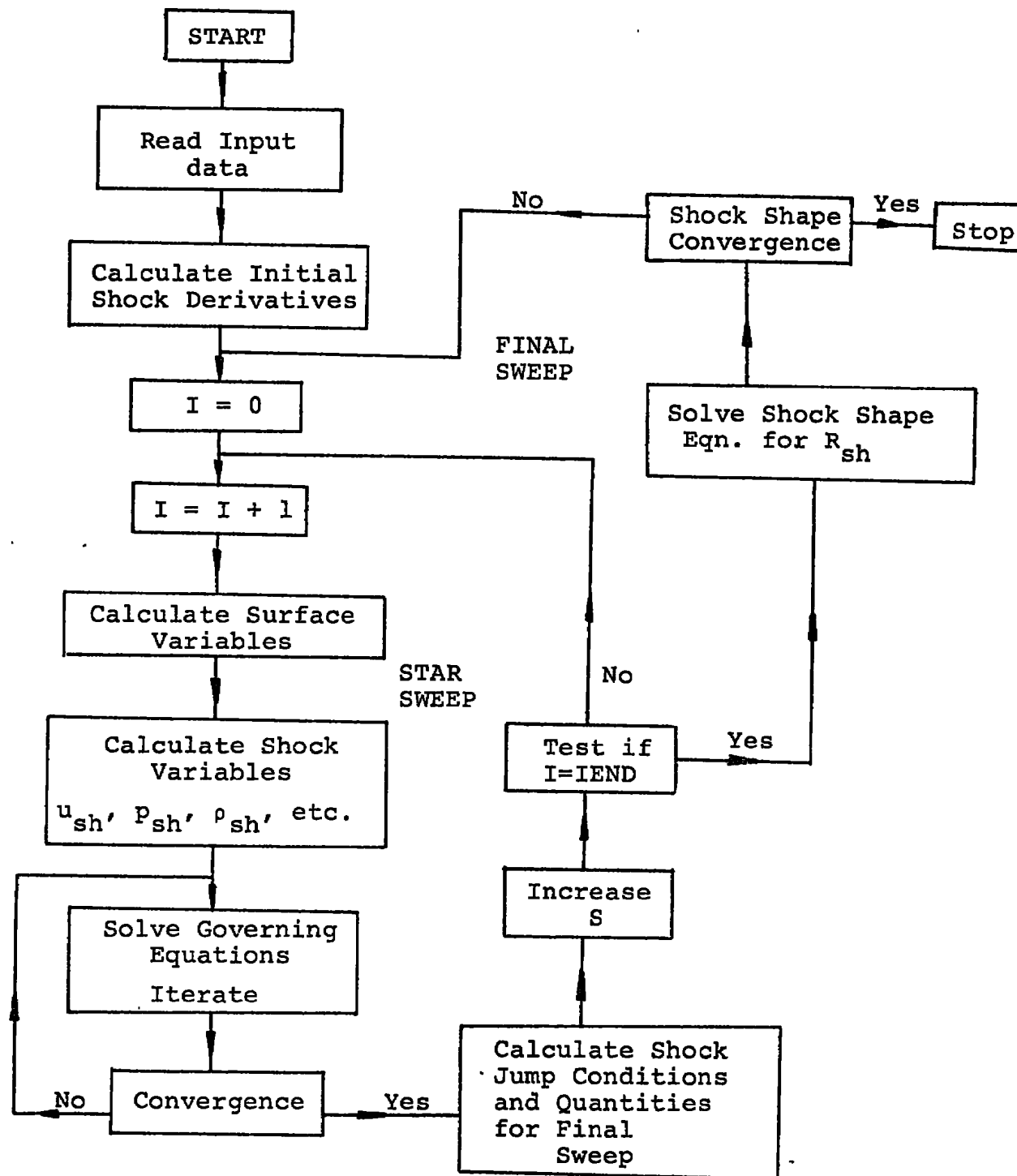
Output Quantities

MINF	Free stream Mach number.
TW/TO	Wall to stagnation temperature ratio, t_w/t_o .
EPS	Defined as, $[\mu^*(u_\infty^{*2}/C_p^*)/\rho_\infty^* U_\infty^* a^*]^{1/2}$.
REY(INF)	Free stream Reynolds number, Re_∞ .
S	ξ , surface distance.
X	Axial distance measured from nose.
RSH	Shock distance measured from axis.
NSH	Shock stand off distance normal to body surface.
XSH	Shock axial distance measured from body nose point.
R	Normal distance to the body surface from the axis.
NSHP	$dn_s/d\xi$.
USH	u-component of velocity behind the shock.
VSH	v-component of velocity behind the shock.
TSH	Temperature behind the shock.
RSH	Density behind the shock.
PSH	Pressure behind the shock.
USP	$du_{sh}/d\xi$.
VSP	$dv_{sh}/d\xi$.

TSP	$d\tau_{sh}/d\xi.$
RSP	$d\rho_{sh}/d\xi.$
PSP	$d\rho_{sh}/d\xi.$
PWALL	Pressure at the wall, $p^*/\rho_\infty^* U_\infty^{*2}.$
PW/PO	Pressure ratio at the wall
CF	Skin friction coefficient, $2\tau_w^*/\rho_\infty^* U_\infty^{*2}.$
STAN	Stanton number, $q_w/(H_o-H_w).$
HEAT	Wall heat transfer, $q_w^*/(\rho_\infty^* U_\infty^{*3}).$

List of Subroutines

<u>Name</u>	<u>Function</u>
DERIV	Calculates the derivatives of the initial shock shape.
GEOM	Calculates body geometry for any given longitudinal location, s.
SHVALS	Calculates properties behind the shock.
MANISH	Utilizes shock jump conditions to evaluate new shock shape, final sweep.
PUSHPA	Evaluates further shock quantities using new shock shape.
PEQSO	Solves tridiagonal difference equation.
BOUND	Provides initial coefficient for derivative boundary conditions.
BOUND1	Provides initial coefficients for derivative boundary condition at $s=0$ on the shock for final sweep.

Flow Chart for Sphere Cone Program:

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0001      IMPLICIT REAL*8 (A-H, O-Z)
0002      COMMON/MAIN1/
1          T1(201),T2(201),TC(201),U1(201),U2(201),UC(201),
1          V1(201),V2(201),VC(201),P1(201),P2(201),PC(201),
2          T1N(201),T2N(201),TCN(201),U1N(201),U2N(201),UCN(201),
3          T1NN(201),T2NN(201),U1NN(201),U2NN(201),PCN(201),
4          AA(201),BB(201),RVISC(201),CON(201),VISC(201),
5          RCON(201),CPST(201),RNSH(201),RCSF(201),
6          R1(201),R2(201),RC(201),P2N(201),P0(201),P0N(201),
7          PS(201),V2N(201),VS(201),C01(201),C02(201),PE(201),
8          P1N(201),P21(201),P22N(201),P22(201),PFAC(201),P21N(201),
9          ,VG(201),VGN(201),P33(201),P33N(201),VGS(201),
1         V0(201),V0N(201),XM(201),PITD(201),UCNN(201)
0003      COMMON /PEQS/ DS,DN(201),IM,IE,A1(201),A2(201),A3(201),A4(201),
1         XN(202)
0004      COMMON /INSH/ CONO , GAM , S , UPSH , XNS ,
1         EPS , RMAC , TPSH , VISCO
0005      DIMENSION P2G(201)
0006      DIMENSION P13N(202)
0007      DIMENSION PFAM(210)
0008      DIMENSION C12(210),C11(210)
0009      DIMENSION CNS2P(40),CNS2PP(40)
0010      DIMENSION V1G(210)
0011      DIMENSION V2G(210)
0012      DIMENSION CNS2(110)
0013      DIMENSION VCD1(201,5)
0014      DIMENSION VCD1(201,5)
0015      DIMENSION YNSH(110),YNSP(110),YNSPP(110)
0016      COMMON/PUSHY/ DERIV1,THMAX
0017      COMMON/OUTSH/ PPS , RRS , TTS , UUS1 , VVS ,
1         PPS1 , RRS1 , TTS1 , UUS2 , VVS1 ,
2         PSP , RRS2 , TSP , USP , VVS2 ,
3         PPS2 , RSP , TTS2 , UUS , VSP
0018      COMMON/BASU/ XS3 , CONE
0019      COMMON/KINNI/ XNSH(110),XNSP(110),XNSPP(110)
0020      COMMON/MANIS/ AXSH(110),AXSP(110),AXSPP(110)
0021      COMMON/MANU/EE1,FF1,IEND,IEND1,AAA1(110),AAA2(110),AAA3(110)
1,AAA4(110)
0022      COMMON/CON/ NJNC,NJ1,RUMP
0023      COMMON/MAIN2/ CNS,ALP,CONP ,AKK1,ALP3,PHI3,VVM,AKK2,AKK3,X1SP
1,PHI,CSF2,RS2,TKK1,TKK2,TKK3
0024      COMMON/MN2/ RE1A,RE1B,RE2A,RE2B,RE3A,RE3B,YNPPJ,YNSPJ,YNSHJ,DT,
1 RSH1,I,J,XNSPJ,AMM3(202),RSH
0025      DATA BLNK /' ',BNO/'NO'/
C COMPUTER CODE FOR SPHERICALLY BLUNTED CONE USING FULL SHOCK LAYER
C EQUATIONS (PROGRAMMED BY B.N.SRIVASTAVA)
0026      READ(5,30) DT,THETA,THETA1,WW,XFACT
0027      READ(5,31) J,NJNC,IE,IEND
0028      READ(5,32) RMAC,B0,REYN,TEMP,GAM,SIGM
0029      30 FORMAT(5F10.6)
0030      31 FORMAT(4I10)
0031      32 FORMAT(6F12.4)
0032      WRITE(6,33) DT, THETA,THETA1,WW,XFACT,GAM,SIGM,J,NJNC

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0033      33      FORMAT(1H1,45X,10HINPUT DATA,/
17F12.6,2I10)
0034          DTHETA=3.50D0
0035          NTIME1=0
0036          NJ1=NJNC-1
0037          NJ2=NJ1-1
0038          NJNC1=NJNC+1
0039          THMAX=THETA*3.14159225D0/180.0D0
0040          SMAX=3.1415926535897932D0/2.0D0-THMAX
0041          ANJ1=NJ1
0042          DS=SMAX/(ANJ1-0.50D0)
0043          RUMP=1.0D0
0044          RSH1I=0.0D0
0045          THSL=1.0000
0046          THINI=-1.0D0
0047          AFULL=-1.0D0
0048          ALSL=1.0000D0
0049          CONVER=-1.0D0
0050      36      CONTINUE
0051          THIN=THINI
0052      35 CONTINUE
0053          TIME=0.0D0
0054          NTIME=0
0055          SWFAC=-1.00D0
0056          SSFAC=-1.00D0
0057          CALL DERIV(DS,IEND,IEND1,YNSH,YNSP,YNSPP)
0058          THMAX=THETA*3.14159225D0/180.0D0
0059          SMAX=3.1415926535897932D0/2.0D0-THMAX
0060          ANJ1=NJ1
0061          DS=SMAX/(ANJ1-0.50D0)
0062          NJ2=NJ1-1
0063          NJNC1=NJNC+1
0064      77      CONTINUE
0065          IF(TIME.GT.35.0D0)      DT=100.0D0
0066          THMAX=THETA*3.14159225D0/180.0D0
0067          SMAX=3.1415926535897932D0/2.0D0-THMAX
0068          DS=SMAX/(ANJ1-0.50D0)
0069      778     CONTINUE
0070          NTIME1=NTIME1+1
0071          DO 777      N=1,IEND1
0072              CNS2(N)=YNSH(N)
0073              CNS2P(N)=YNSP(N)
0074              CNS2PP(N)=YNSPP(N)
0075      777     CONTINUE
0076          DERIV1=-1.0D0
0077          XNS0=XNSH(1)
0078          NTIME=NTIME+1
0079          IM=IE-1
0080          XN(1)=0.0D0
0081          DO 15 N=1,IE
0082              IF(XN(N).LE.1.0D0)      DY=0.0350D0
0083              IF(XN(N).LE.0.6499999D0)      DY=0.0150D0
0084              IF(XN(N).LE.0.0499999D0)      DY=0.0010D0

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0085      DN(N)=DY
0086      15 XN(N+1)=XN(N)+DN(N)
0087      NITER=0
0088      RSH=0.0D0
0089      RSH1=RSH1 I
0090      UUS0=0.0
0091      URSH=0.0
0092      UPSH=0.0D0
0093      TPSH=0.0D0
0094      VISCO=0.0D0
0095      CONO=0.0D0
0096      ASL=1.2304*(2.0-THSL)/THSL
0097      BSL=1.1750*(2.0-THSL)/THSL
0098      CSL=2.3071D0*(2.0D0-ALSL)/ALSL
0099      XNS=XNS0
0100      XNS1=XNS
0101      DS2=DS/2.0D0
0102      CK=1.0D0
0103      CSF=0.0D0
0104      SIF=1.0D0
0105      RS=0.0D0
0106      RS2=0.0D0
0107      XB=0.0
0108      CDF=0.0
0109      CDP=0.0
0110      CDP1=0.0
0111      CDP2=0.0
0112      CDF1=0.0
0113      CDF2=0.0
0114      CDPD=0.0
0115      CDFD=0.0
0116      CNS=(XNS1+XNS)/2.0
0117      P0IP=((GAM+1.)*RMAC*RMAC /2.0)**(GAM/(GAM-1.0))/(GAM*RMAC*RMAC*
1      (2.0*GAM*RMAC*RMAC/(GAM+1.0)-(GAM-1.0)/(GAM+1.0))**{1.0/(GAM-1.0)})
0118      TW=B0*(1.0D0/((GAM-1.0D0)*RMAC*RMAC)+0.50D0)
0119      TB=TW*((GAM-1.0D0)*RMAC*RMAC*TEMP)
0120      CONP=198.6/((GAM-1.0)*RMAC*RMAC*TEMP)
0121      VISRA=(1.0+CONP)*(1.0/((GAM-1.0)*RMAC*RMAC))*1.5/(1.0/((GAM-1.0)*
1      RMAC*RMAC)+CONP)
0122      EPS = 1.0 / DSQRT(REYIN*VISRA)
0123      CALL SHVALS(1.0D0,0.0D0,1.0D0,0.0D0,TTSO,VVS0,UUS0,PPS0,1)
0124      TTS=TTSO
0125      DO 100 N=1,IE
0126      RNSH(N)=CNS/(1.0+CK*CNS*XN(N))
0127      RCSF(N)=CNS/(1.0+CK*CNS*XN(N))
0128      U1(N)=XN(N)
0129      U2(N)=XN(N)
0130      U1N(N)=1.0
0131      U2N(N)=1.0
0132      U1NN(N)=0.0
0133      UC(N)=XN(N)
0134      UCN(N)=1.0
0135      V1(N)=XN(N)

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0136      V2(N)=XN(N)
0137      VC(N)=XN(N)
0138      T1(N)=1.0-((1.0-XN(N))*(1.0-TW/TTS0))
0139      T2(N)=T1(N)
0140      T1N(N)=1.0-TW/TTS0
0141      T2N(N)=T1N(N)
0142      T1NN(N)=0.0
0143      TC(N)=T1(N)
0144      TCN(N)=T1N(N)
0145      VISC(N)=(TTS+CONP)*TC(N)**1.5/(TTS*TC(N)+CONP)
0146      RVISC(N)=(TTS*TC(N)+3.0*CONP)/(2.0*TC(N)*(TTS*TC(N)+CONP))*TCN(N)
0147      CON(N)=VISC(N)
0148      RCON(N)=RVISC(N)
0149      P1(N)=1.0
0150      P2(N)=1.0
0151      PC(N)=1.0
0152      PS(N)=0.0
0153      P0(N)=1.0
0154      P0N(N)=0.0
0155      R1(N)=P1(N)/T1(N)
0156      R2(N)=R1(N)
0157      RC(N)=R1(N)
0158      PFAC(N)=1.0
0159      PCN(N)=0.0
0160      P1N(N)=0.0
0161      100 P2N(N)=0.0
0162      AA(1)=0.0
0163      BB(1)=0.0
0164      VISCO=(1.0+CONP)*TTS**1.5/(TTS+CONP)
0165      CONO=VISCO/SIGM
0166      DO 5000 I=1,IEND
0167      CRNI=1.0D0
0168      IF(I.LT.60)      W=0.8
0169      IF(I.LT.9)      W=0.60D0
0170      YI=I
0171      S=(YI-1.0D0)*DS
0172      CALL GEOM(S,DS2,RS2,CK2,CSF2,SIF2,XB2)
0173      PHI = DARCOS(CSF2)
0174      PHI2=PHI
0175      IF(I.EQ.1)      PHI1= 3.1415926535897932D0/2.0D0
0176      PHIS2=(PHI2-PHI1)/DS*2.0D0
0177      IF(I.EQ.1)      PHIS=PHIS2
0178      IF(CONE.LT.0.0D0)      PHIS2=-1.0D0
0179      IF(CONE.LT.0.0D0)      PHIS=-1.0D0
0180      IF(CONE.GT.0.0D0)      PHIS2=0.0D0
0181      IF(CONE.GT.0.0D0)      PHIS=0.0D0
0182      DXDS1=(AXSP(I)+AXSP(I+1))/2.0D0
0183      X1SP=XNSP(I)
0184      XNSPM=(XNSP(I)+XNSP(I+1))/2.0D0
0185      ALP=DATAN((YNSP(I)+YNSP(I+1))/(2.0D0*DXDS1))
0186      IF(RUMP.LT.0.0D0)      GO TO 305
0187      IF(I.EQ.NJ1)XNSPM=(3.0D0*XNSP(NJ1)-XNSP(NJ2))/2.0D0
0188      IF(I.EQ.NJ1)      AXSPJ=(3.0D0*AXSP(NJ1)-AXSP(NJ2))/2.0D0

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0189      IF(I.EQ.NJ1)  YNPPJ=(3.0D0*YNSPP(NJ1)-YNSPP(NJ2))/2.0D0
0190      IF(I.EQ.NJ1)  YNSHJ=(3.0D0*YNSH(NJ1)-YNSH(NJ2))/2.0D0
0191      IF(I.EQ.NJ1)  XNSHJ=(3.0D0*XNSH(NJ1)-XNSH(NJ2))/2.0D0
0192      IF(I.EQ.NJ1)  YNSPJ=(3.0D0*YNSP(NJ1)-YNSP(NJ2))/2.0D0
0193      IF(I.EQ.NJ1)  ALP=DATAN(YNSPJ/AXSPJ)
0194      305      CONTINUE
0195      IF(I.EQ.1)  ALP=(22.0D0/14.0D0+DATAN(YNSP(I+1)/AXSP(I+1)))/2.0
0196      SP = DSIN(ALP)
0197      CP = DCOS(ALP)
0198      SPB=SP*SIF2+CP*CSF2
0199      CPB=CP*SIF2-SP*CSF2
0200      2009 CONTINUE
0201      CALL SHVALS ( SP, CP, SPB, CPB, TTSH, VRSH, URSH, PPSH, 2)
C*****
0202      IF(I.EQ.1)  ALP3= 3.1415926535897932D0/2.0D0
0203      IF(I.EQ.1)  PHI3= 3.1415926535897932D0/2.0D0
0204      BLP=ALP3
0205      PHP=PHI3
0206      AK=TTS/PPS
0207      AR=2.0D0*GAM*RMAC*RMAC*DSIN(BLP)**2.0D0-(GAM-1.0D0)
0208      AR1=2.0D0*GAM*GAM*RMAC*RMAC*DSIN(2.0D0*BLP)/(AR*(GAM+1.0D0)**1.0
1D0)
0209      AR2=4.0D0*GAM*DCOS(BLP)/((GAM+1.0D0)*RMAC**2.0D0*AR*DSIN(BLP)**
13.0D0)
0210      AR3=4.0D0*GAM**3.0D0*RMAC**4.0D0*DSIN(2.0D0*BLP)*DSIN(BLP)**2.0D0
1/(AR**2.0D0*(GAM+1.0D0)**1.0D0)
0211      AR4=2.0D0*GAM*RMAC**2.0D0*DSIN(2.0D0*BLP)/(AR*AR)*(GAM*(GAM+1.0D0
1)/(GAM-1.0D0)-2.0D0*GAM*(GAM-1.0D0)/(GAM+1.0D0))
0212      AR5=4.0D0*GAM*GAM*DSIN(2.0D0*BLP)/((GAM+1.0D0)*AR*AR*DSIN(BLP)**2.
10D0)
0213      ARR1=AR1+AR2-AR3-AR4+AR5
0214      AKK1=-DSIN(2.0D0*BLP-PHP)*(1.0D0-(GAM-1.0D0)*AK/GAM)
1+DSIN(BLP)*DSIN(BLP-PHP)*(GAM-1.0D0)/GAM*ARR1
0215      AK11=+DCOS(2.0D0*BLP-PHP)*(1.0D0-(GAM-1.0D0)*AK/GAM)
1-DSIN(BLP)*DCOS(BLP-PHP)*(GAM-1.0D0)/GAM*ARR1
0216      AKK2=DCOS(BLP)*DSIN(BLP-PHP)-(GAM-1.0D0)/GAM*AK*DSIN(BLP)*DCOS(BLP
1-PHP)
0217      AKK2=-DCOS(BLP)*DCOS(BLP-PHP)-(GAM-1.0D0)/GAM*AK*DSIN(BLP)*DSIN
1(BLP-PHP)
0218      AK6=2.0D0*GAM/((GAM+1.0D0)**2.0D0)*DSIN(2.0D0*BLP)
0219      AK7=4.0D0*DCOS(BLP)/((GAM+1.0D0)**2.0D0*RMAC**4.0D0*DSIN(BLP)**3.
10D0)
0220      AKK6=AK6+AK7
0221      AKK3=2.0D0/(GAM+1.0D0)*DSIN(2.0D0*BLP)
0222      IF(I.EQ.1)  GO TO 302
0223      DALDS=(DCOS(ALP3-PHI3)**2.0D0/((1.0D0+CK*CNS)*DCOS(PHI3)))
1*(YNSPP(I)-((RSH-YNSH(I))/DT)*2.0D0)-YNSP(I)*CK*DSIN(ALP3*2.0D0
2-2.0D0*PHI3)/((1.0D0+CK*CNS)*DCOS(PHI3))
0224      302      CONTINUE
0225      IF(I.EQ.1)  DALDS=((XNSPP(1)-((CNS-XNSH(1))/DT)*2.0D0)/(1.0D0+
1CNS)-1.0D0)
0226      USP=AKK1*DALDS+AKK2*PHI3
0227      VSP=AK11*DALDS+AKK2*PHI3

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0228      PSP=AKK3*DALDS
0229      TSP=AKK6*DALDS
0230      RSP=(GAM/(GAM-1.0D0))* (PSP*TTS-TSP*PPS)/(TTS*TTS)
0231      IF(I.EQ.1)      PSP=0.0D0
0232      IF(I.EQ.1)      TSP=0.0D0
0233      IF(I.EQ.1)      VSP=0.0D0
0234      37      CONTINUE
0235      TLP=ALP
0236      THP=PHI
0237      TK=TT S2/PPS2
0238      TR=2.0D0*GAM*RMAC*RMAC*DSIN(TLP)* DSIN(TLP)-(GAM-1.0D0)
0239      TR1=2.0D0*GAM*GAM*RMAC*RMAC*DSIN(2.0D0*TLP)/(TR*(GAM+1.0D0)**1.0
1D0)
0240      TR2=4.0D0*GAM*DCOS(TLP)/((GAM+1.0D0)*RMAC**2.0D0*TR*DSIN(TLP)*
1DSIN(TLP)*DSIN(TLP))
0241      TR3=4.0D0*GAM**3.0D0*RMAC**4.0D0*DSIN(2.0D0*TLP)*DSIN(TLP)**2.0D0
1/(TR**2.0D0*(GAM+1.0D0)**1.0D0)
0242      TR4=2.0D0*GAM*RMAC**2.0D0*DSIN(2.0D0*TLP)/(TR*TR)*(GAM*(GAM+1.0D0
1)/(GAM-1.0D0)-2.0D0*GAM*(GAM-1.0D0)/(GAM+1.0D0))
0243      TR5=4.0D0*GAM*GAM*DSIN(2.0D0*TLP)/((GAM+1.0D0)*TR*TR*DSIN(TLP)**2.
10D0)
0244      TRR1=TR1+TR2-TR3-TR4+TR5
0245      TKK1=-DSIN(2.0D0*TLP-THP)*((1.0D0-(GAM-1.0D0)*TK/GAM)
1+DSIN(TLP)*DSIN(TLP-THP)*(GAM-1.0D0)/GAM*TRR1
0246      TK11=+DCOS(2.0D0*TLP-THP)*((1.0D0-(GAM-1.0D0)*TK/GAM)
1-DSIN(TLP)*DCOS(TLP-THP)*(GAM-1.0D0)/GAM*TRR1
0247      TKK2=DCOS(TLP)*DSIN(TLP-THP)-(GAM-1.0D0)/GAM*TK*DSIN(TLP)*DCOS(TLP
1-THP)
0248      TKKK2=-DCOS(TLP)*DCOS(TLP-THP)-(GAM-1.0D0)/GAM*TK*DSIN(TLP)*DSIN
1(TLP-THP)
0249      DALDS1=(DCOS(TLP-THP)**2.0D0/((1.0D0+CK2*XNS)*DCOS(THP)))
1*((YNSPP(I)+YNSPP(I+1))/2.0D0-((RSH1-((YNSH(I)+YNSH(I+1))/2.0D0))
2/DT)*2.0D0)-CK2*DSIN(TLP*2.0D0-THP*2.0D0)/((1.0D0+CK2*XNS)*
2DCOS(THP))*((YNSP(I)+YNSP(I+1))/2.0D0)
0250      IF(RUMP.LT.0.0D0)      GO      TO      306
0251      IF(I.EQ.NJ1)
1 DALDS1=(DCOS(TLP-THP)**2.0D0/((1.0D0+CK*XNS)*DCOS(THP)))
2*(YNPPJ      -((RSH1-YNSHJ)
3/DT)*2.0D0)-CK2*DSIN(TLP*2.0D0-THP*2.0D0)/((1.0D0+CK2*XNS)*
4DCOS(THP))*YNSPJ
0252      306      CONTINUE
0253      VSP1=TK11*DALDS1+TKKK2*PHIS2
C*****
0254      IF(I.EQ.1)      VVM=VVS
0255      IF(I.GT.1)      VVM=1.0D0
0256      VISCO=(1.0+CONP)*TTS**1.5/(TTS+CONP)
0257      CONO=VISCO/SIGM
0258      REFAC=RRS*VVM*CNS/(EPS*EPS*VISCO)
0259      VIS2=(TTS2+CONP)*T2(1)**1.5/(TTS2*T2(1)+CONP)
0260      XKSL = VIS2*RRS*VVM*DSQRT((GAM-1.0D0)*TTS2*T2(1)/GAM)/
1      (PPS2*P2(1)*REFAC)
0261      DO 200 N=1,IE
0262      CPST(N)=1.0

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0263       IF (S.GE.0.0001) GO TO 160
0264       PFAC(N)=4.0D0*(P2(N)+(PPS2/PPS0-2.0D0)*P0(N))/(UUS2*DS)
           1      -XNSP(2)*XN(N)*P0N(N)/(2.0D0*UUS2*CNS)
0265       GO TO 200
0266       160  CONTINUE
0267       200  CONTINUE
           C      SOLVE ENERGY EQUATION
0268       DO 500 N=1,IE
0269       A1(N)=REFAC*VISCO*CPST(N)*(UUS*XNSP(1)*RNSH(N)*RC(N)*UC(N)*XN(N)
           1      /(VVM*CNS)-RC(N)*VC(N))/(CONO*CON(N))+RCON(N)+CK*RNSH(N)
           2      +RCSF(N)
0270       A4(N)=-REFAC*VISCO*CPST(N)*UUS*RNSH(N)*RC(N)*UC(N)/(VVM*CONO*
           1      CON(N))
0271       A2(N)=A4(N)*TSP/TTS
0272       500  A3(N)=REFAC*PPS*VISCO*(RNSH(N)*UUS*UUS*UC(N)*PFAC(N)+VVM*VC(N)*
           1      PCN(N))/(TTS*RRS*VVM*CONO*CON(N))+UUS*UUS*VISC(N)*VISCO*
           2      (UCN(N)-CK*RNSH(N)*UC(N))*2/(TTS*CONO*CON(N))
0273       GAMP=GAM+1.0D0
0274       GAMM=GAM-1.0D0
0275       RMACQ=RMAC*RMAC
0276       EPSQ=EPS*EPS
0277       SPQ=SP*SP
0278       FOGQ=4.0D0/(GAMP*GAMP)
0279       DEN=RMACQ*RMACQ*SPQ
0280       CS1=SP*XNS/(EPSQ*CONO)
0281       CS2=-((URSH-CP)**2+FOGQ*GAM*SPQ+(2.0D0/GAMM-FOGQ*GAMM)/RMACQ
           1      -FOGQ/DEN)*0.5D0*SP*XNS/(EPSQ*CONO*TTS2)
0282       IF (SWFAC) 501,501,502
0283       502  CB1=-1./(CSL*XKSL)
0284       CB2=TW/(TTS2*CSL*XKSL)
0285       CALL BOUND(T1NN,T1N,T1,CB1,CB2,E1,F1,CRNI)
0286       GO TO 503
0287       501  E1=0.0
0288       F1=TW/TTS2
0289       503  CALL PEQSO(T1NN,T1N,T1,T2NN,T2N,T2,E1,F1,CRNI,CS1,CS2,SSFAC,1.0,1)
0290       TTS2G=TTS2
0291       IF (SSFAC) 521,521,522
0292       522  TPSH=T2N(IE)
0293       TTS2=T2(IE)*TTS2G
0294       IF (S.GE.0.0001) GO TO 525
0295       TTS1=TTS2
0296       525  TTS=(TTS2+TTS1)/2.0
0297       DO 524 N=1,IE
0298       T2NN(N)=T2NN(N)*TTS2G/TTS2
0299       T2N(N)=T2N(N)*TTS2G/TTS2
0300       524  T2(N)=T2(N)*TTS2G/TTS2
0301       VISCO=(1.0+CONP)*TTS**1.5/(TTS+CONP)
0302       CONO=VISCO/SIGM
0303       REFAC=RRS*VVM*CNS/(EPS*EPS*VISCO)
0304       GO TO 523
0305       521  TPSH=0.
           C      SOLVE S MOMENTUM EQUATION
0306       523  XU25=U2(15)

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0307      XU25=U2(J)
0308      VIS2=(TTS2+CONP)*T2(1)**1.5/(TTS2*T2(1)+CONP)
0309      XKSL = VIS2*RRS*VVM*DSQRT((GAM-1.0D0)*TTS2*T2(1)/GAM)/
          1      (PPS2*P2(1)*REFAC)
0310      DO 540 N=1,IE
0311      R2(N)=P2(N)/T2(N)
0312      IF (S.GE.0.0001) GO TO 541
0313      R1(N)=R2(N)
0314      T1(N)=T2(N)
0315      T1N(N)=T2N(N)
0316      T1NN(N)=T2NN(N)
0317      541  CONTINUE
0318      TC(N)=(T1(N)+T2(N))/2.
0319      TCN(N)=(T1N(N)+T2N(N))/2.
0320      VISC(N)=(TTS+CONP)*TC(N)**1.5/(TTS*TC(N)+CONP)
0321      RVISC(N)=(TTS*TC(N)+3.0*CONP)/(2.0*TC(N)*(TTS*TC(N)+CONP))*TCN(N)
0322      CON(N)=VISC(N)
0323      RCON(N)=RVISC(N)
0324      540  RC(N)=PC(N)/TC(N)
0325      DO 600 N=1,IE
0326      A1(N)=REFAC*(UUS*XNSP(I)*RNSH(N)*RC(N)*UC(N)*XN(N)/(VVM*CNS)
          1      -RC(N)*VC(N))/VISC(N)+RVISC(N)+CK*RNSH(N)+RCSF(N)
0327      A2(N)=-REFAC*(USP*RNSH(N)*RC(N)*UC(N)/VVM+CK*RNSH(N)*RC(N)
          1      *VC(N))/VISC(N)-CK*RNSH(N)*RVISC(N)-(CK*RNSH(N)+
          2      RCSF(N))*CK*RNSH(N)
0328      A3(N)=-REFAC*PPS*RNSH(N)*PFAC(N)/(VISC(N)*RRS*VVM)
0329      600  A4(N)=-REFAC*UUS*RNSH(N)*RC(N)*UC(N)/(VISC(N)*VVM)
0330      CS1=SP*SPB*UUS2*XNS/(EPS*EPS*VISCO*URSH)
          1      -CK2*XNS/(1.+CK2*XNS)
0331      CS2=-SP*XNS*(CP+VVS2*CPB)/(EPS*EPS*VISCO*URSH)
0332      IF (SSFAC) 601,601,602
0333      602  CB1=-1./(ASL*XKSL)-CK2*XNS
0334      CB2=0.
0335      CALL BOUND(U1NN,U1N,U1,CB1,CB2,E1,F1,CRNI)
0336      GO TO 603
0337      601  E1=0.0
0338      F1=0.0
0339      603  CALL PEQSO(U1NN,U1N,U1,U2NN,U2N,U2,E1,F1,CRNI,CS1,CS2,SSFAC,1.,1)
0340      UUS2G=UUS2
0341      IF (SSFAC) 621,621,622
0342      622  UPSH=U2N(IE)-CK2*XNS*U2(IE)/(1.+CK2*XNS)
0343      UUS2=U2(IE)*UUS2G
0344      IF (S.GE.0.0001) GO TO 625
0345      UUS1=-UUS2
0346      625  UUS=(UUS2+UUS1)/2.0
0347      DO 624 N=1,IE
0348      U2NN(N)=U2NN(N)*UUS2G/UUS2
0349      U2N(N)=U2N(N)*UUS2G/UUS2
0350      624  U2(N)=U2(N)*UUS2G/UUS2
0351      GO TO 623
0352      621  UPSH=0.
0353      C  SOLVE MASS CONSERVATION EQUATION
          623  CONTINUE

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0354      DO 640 N=1,IE
0355      IF (S.GE.0.0001) GO TO 641
0356      U1(N)=U2(N)
0357      U1N(N)=U2N(N)
0358      U1NN(N)=U2NN(N)
0359      641 UC(N)=(U1(N)+U2(N))/2.
0360      UCNN(N)=(U1NN(N)+U2NN(N))/2.0D0
0361      640 UCN(N)=(U1N(N)+U2N(N))/2.
0362      DO 6055 N=1,IE
0363      P2G(N)=P2(N)
0364      V1G(N)=VC(N)
0365      V2G(N)=V2(N)
0366      6055 CONTINUE
0367      XV10=P2(J)
0368      XV50=T2(J)
0369      DO 700 N=2,IE
0370      AA(N)=AA(N-1)+DN(N-1)*(R2(N-1)*U2(N-1)+R2(N)*U2(N))/2.
0371      700 BB(N)=BB(N-1)+DN(N-1)*(R2(N-1)*U2(N-1)*XN(N-1)
1      +R2(N)*U2(N)*XN(N))/2.
0372      IF (S.GE.0.0001) GO TO 705
0373      AIA=8.*BB(IE)*RRS2*UUS2*CSF2/DS-DS
0374      BIB=4.*AA(IE)*RRS2*UUS2*RS2/DS-DS
0375      CIC=-DS
0376      ROT=BIB*BIB-AIA*CIC
0377      XNS = (-BIB + DSQRT(ROT)) / AIA
0378      X1R=XNS
0379      DO 701 N=1,IE
0380      XNR=XNS
0381      C12(N)=C02(N)
0382      C02(N)=XNR*(RS2*AA(N)+XNR*CSF2*BB(N))*RRS2*UUS2
0383      VC(N)=8.*XNS*(AA(N)*RS2+XNS*BB(N)*CSF2)*RRS2*UUS2/(DS*DS*(1.+
1      XNS*XN(N))*2 *RC(N))
0384      V2(N)=VC(N)
0385      701 CONTINUE
0386      GO TO 711
0387      705 AIA=BB(IE)*CSF2*RRS2*UUS2
0388      BIB=AA(IE)*RS2*RRS2*UUS2/2.
0389      CIC=-C01(IE)+(RS+CNS*CSF)*((1.+CK*CNS)*RRS*VVS-XNSP(1))*RRS*UUS)*DS
0390      ROT=BIB*BIB-AIA*CIC
0391      XNS = (-BIB + DSQRT(ROT)) / AIA
0392      CIC1=-C11(IE)+(RS2+X1R*CSF2)*((1.+CK2*X1R)*RRS2*VVS2-XNSPM*RRS2
1      *UUS2)*DS
0393      ROT1=BIB*BIB-AIA*CIC1
0394      XN1S=(-BIB+DSQRT(ROT1))/AIA
0395      DO 710 N=1,IE
0396      XNR=XNS
0397      C02(N)=XNR*(RS2*AA(N)+XNR*CSF2*BB(N))*RRS2*UUS2
0398      VI=(C02(N)-C01(N))/DS
0399      VC(N)=-VI/(RRS*VVM*RC(N))*((1.+CK*CNS*XN(N))*((RS+CNS*XN(N)*CSF))+
1      XNSP(1)*XN(N)*UUS*UC(N)/(VVM*(1.+CK*CNS*XN(N))))
0400      X1R=XN1S
0401      C12(N)=X1R*(RS2*AA(N)+X1R*CSF2*BB(N))*RRS2*UUS2
0402      VM=(C12(N)-C11(N))/DS

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0403      V2(N)=-VM/(RRS2*VVM*R2(N)*(1.+CK2*X1R*XN(N))*(RS2+X1R*XN(N)*CSF2)
          2)+XNSPM*XN(N)*UUS2*U2(N)/(VVM*(1.+CK2*X1R*XN(N)))
0404      145 CONTINUE
0405      710 CONTINUE
0406      711 IF (S.GE.0.0001) GO TO 715
0407      XNS1=XNS
0408      715 CNS=(XNS1+XNS)/2.
0409      RSH1=RS2+XNS*CSF2
0410      IF(I.EQ.1)      RSH1I=RSH1
0411      XSH=XB-CNS*SIF
0412      RSH=RS+CNS*CSF
0413      IF(THIN.GT.0.000)      GO TO 9116
0414      6020 CONTINUE
0415      DO 9117      N=1,IM
0416      VC(N)=(W*V1G(N)+(1.-W)*VC(N))
0417      V2(N)=(W*V2G(N)+(1.-W)*V2(N))
0418      9117 CONTINUE
0419      9116 CONTINUE
0420      IF (THIN.GE.0.0) GO TO 716
0421      VVS2G=VVS
0422      VVS2G=VVS2
0423      VPG=VSP1
0424      716 CONTINUE
0425      DO 712 N=1,IE
0426      RNSH(N)=CNS/(1.+CK*CNS*XN(N))
0427      IF (S.GE.0.0001) GO TO 713
0428      V1(N)=VC(N)
0429      RCSF(N)=CNS/(1.+CK*CNS*XN(N))
0430      GO TO 714
0431      713 RCSF(N)=CSF*CNS/(RS+CNS*XN(N)*CSF)
0432      714 CONTINUE
0433      VS(N)=(V2(N)-V1(N))/DS
0434      IF (THIN.GE.0.0) GO TO 717
0435      VG(N)=V2(N)
0436      VGS(N)=VS(N)
0437      IF(NTIME.EQ.1)      GO TO 718
0438      IF(I.LE.2)      VGS(N)=(VCDI(N,I+1)-VCDI(N,I))/DS
0439      IF(I.EQ.1)      VGS(N)=(VCDI(N,I+1)-VCDI(N,I))/DS
0440      718 CONTINUE
0441      IF(I.EQ.1)      V0(N)=VC(N)
0442      717 CONTINUE
0443      712 CONTINUE
0444      DO 720 N=2,IM
0445      IF (THIN.GE.0.0) GO TO 720
0446      V0N(N)=(DN(N-1)*V0(N+1)/DN(N)-DN(N)*V0(N-1)/DN(N-1))/(DN(N)+
          1 DN(N-1))+((DN(N)-DN(N-1))*V0(N)/(DN(N)*DN(N-1))
0447      VGN(N)=(DN(N-1)*VG(N+1)/DN(N)-DN(N)*VG(N-1)/DN(N-1))/(DN(N)+
          1 DN(N-1))+((DN(N)-DN(N-1))*VG(N)/(DN(N)*DN(N-1))
0448      720 V2N(N)=(DN(N-1)*V2(N+1)/DN(N)-DN(N)*V2(N-1)/DN(N-1))/(DN(N)+
          1 DN(N-1))+((DN(N)-DN(N-1))*V2(N)/(DN(N)*DN(N-1))
0449      IF (THIN.GE.0.0) GO TO 725
0450      V0N(IE)=V0(IE)*(DN(IM-1)+2.*DN(IM))/(DN(IM)*(DN(IM)+DN(IM-1)))
          1 -V0(IE-1)*(DN(IM-1)+DN(IM))/(DN(IM)*DN(IM-1))

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      2      +V0(IE-2)*DN(IM)/(DN(IM-1)*(DN(IM)+DN(IM-1)))
0451  VGN(IE)=VG(IE)*(DN(IM-1)+2.*DN(IM))/(DN(IM)*(DN(IM)+DN(IM-1)))
      1      -VG(IE-1)*(DN(IM-1)+DN(IM))/(DN(IM)*DN(IM-1))
      2      +VG(IE-2)*DN(IM)/(DN(IM-1)*(DN(IM)+DN(IM-1)))
0452  VON(1)=-V0(1)*(DN(2)+2.*DN(1))/(DN(1)*(DN(1)+DN(2)))
      1      +V0(2)*(DN(2)+DN(1))/(DN(1)*DN(2))
      2      -V0(3)*DN(1)/(DN(2)*(DN(1)+DN(2)))
0453  VGN(1)=-VG(1)*(DN(2)+2.*DN(1))/(DN(1)*(DN(1)+DN(2)))
      1      +VG(2)*(DN(2)+DN(1))/(DN(1)*DN(2))
      2      -VG(3)*DN(1)/(DN(2)*(DN(1)+DN(2)))
0454  725 CONTINUE
0455  V2N(IE)=V2(IE)*(DN(IM-1)+2.*DN(IM))/(DN(IM)*(DN(IM)+DN(IM-1)))
      1      -V2(IE-1)*(DN(IM-1)+DN(IM))/(DN(IM)*DN(IM-1))
      2      +V2(IE-2)*DN(IM)/(DN(IM-1)*(DN(IM)+DN(IM-1)))
0456  V2N(1)=-V2(1)*(DN(2)+2.*DN(1))/(DN(1)*(DN(1)+DN(2)))
      1      +V2(2)*(DN(2)+DN(1))/(DN(1)*DN(2))
      2      -V2(3)*DN(1)/(DN(2)*(DN(1)+DN(2)))
C      SOLVE N MOMENTUM EQUATION
0457  P21N(IE)=RRS2*UUS2**2 *CK2*XNS/(PPS2*(1.+CK2*XNS))
0458  P21(IE)=1.0
0459  P2N(IE)=P21N(IE)
0460  P2(IE)=1.
0461  RC(IE)=1.0
0462  IF (THIN.GE.0.0) GO TO 750
0463  IF(I.EQ.1)
1P33N(IE)=-RRS2*VVS2G*VVS2G*((1.-UUS2*XNSPM/(VVS2G*(1.+CK2*XNS)))*
1      VGN(IE)+UUS2*XNS*VPG/VVS2G/(VVS2G*(1.+CK2*XNS)))/PPS2
0464  IF(I.GT.1)
1P33N(IE)=-RRS2*((VVS2G-UUS2*XNSPM/(1.0D0+CK2*XNS))*VGN(IE)
2+UUS2*XNS/(1.0D0+CK2*XNS)*VGS(IE))/PPS2
0465  P33(IE)=0.0
0466  P2N(IE)=P2N(IE)+P33N(IE)
0467  750 CONTINUE
0468  PC(IE)=1.
0469  PCN(IE)=(P1N(IE)+P2N(IE))/2.
0470  IF (S.LE.0.0001) GO TO 755
0471  PFAC(IE)=(-XNSP(I)*PCN(IE)/CNS+PSP/PPS)/UUS
0472  755 CONTINUE
0473  PE(IE)=1.
0474  PS(IE)=0.
0475  R2(IE)=1.
0476  IF (S.GE.0.0001) GO TO 800
0477  CALL SHVALS(1.0D0,0.00D0,1.0D0,0.00D0,TT50,VVS0,UUS0,PPS0,1)
0478  P0N(IE)=0.0
0479  IF (THIN.GE.0.0) GO TO 760
C      P0N(IE)=VVS0(1)*VON(IE)/PPS0
      P0N(IE)=VVS *VON(IE)/PPS0
0480  760 CONTINUE
0481  P0(IE)=1.0
0482  P1(IE)=1.
0483  P1N(IE)=P2N(IE)
0484  R1(IE)=1.
0485  V1(IE)=1.
0486

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0487      800 KON=IM
0488      DO 810 N=1,IM
0489      P21N(KON)=RRS2*UUS2**2 *CK2*XNS*R2(KON)*U2(KON)**2 /(PPS2*(1.+CK2*
1          XNS*XN(KON)))
0490      P21(KON)=P21(KON+1)-DN(KON)*(P21N(KON+1)+P21N(KON))/2.
0491      P2N(KON)=P21N(KON)
0492      P22(KON)=0.000
0493      P2(KON)=P21(KON)
0494      IF (THIN.GE.0.0) GO TO 805
0495      IF (I.EQ.1)
1 P33N(KON)=-RRS2*VVS2G*VVS2G*((R2(KON)*VG(KON)-R2(KON)*U2(KON)*UUS2
1          *XNSPM*XN(KON))/(VVS2G*(1.+CK2*XNS*XN(KON))))*VGN(KON)
2          +UUS2*XNS*R2(KON)*U2(KON)*(VGS(KON)+VPG*VG(KON)/VVS2G)
3          /(VVS2G*(1.+CK2*XNS*XN(KON))))/PPS2
0496      IF (I.GT.1)
1 P33N(KON)=-RRS2          *((R2(KON)*VG(KON)-R2(KON)*U2(KON)*UUS2
1          *XNSPM*XN(KON))/(1.000*(1.+CK2*XNS*XN(KON))))*VGN(KON)
2          +UUS2*XNS*R2(KON)*U2(KON)*(VGS(KON))
3          /(1.000*(1.+CK2*XNS*XN(KON))))/PPS2
0497      P33(KON)=P33(KON+1)-DN(KON)*(P33N(KON+1)+P33N(KON))/2.
0498      P2N(KON)=P2N(KON)+P33N(KON)
0499      P2(KON)=P2(KON)+P33(KON)
0500      IF (THIN.GT.0.000) GO TO 9222
0501      P2(KON)=(W*P2G(KON)+(1.-W)*P2(KON))
0502  9222 CONTINUE
0503  805 CONTINUE
0504      R2(KON)=P2(KON)/T2(KON)
0505      IF (S.GE.0.0001) GO TO 801
0506      P0N(KON)=0.0
0507      IF (THIN.GE.0.0) GO TO 807
0508      P0N(KON)= VVS *P0(KON)*V0(KON)*V0N(KON)/(PPS0*T2(KON))
0509  807 CONTINUE
0510      P0(KON)=P0(KON+1)-DN(KON)*(P0N(KON+1)+P0N(KON))/2.
0511      P1(KON)=P2(KON)
0512      P1N(KON)=P2N(KON)
0513      R1(KON)=R2(KON)
0514      V1(KON)=V2(KON)
0515  801 PE(KON)=P21(KON)+P22(KON)
0516      PC(KON)=(P1(KON)+P2(KON))/2.
0517      RC(KON)=PC(KON)/TC(KON)
0518      PCN(KON)=(P1N(KON)+P2N(KON))/2.
0519      PS(KON)=(P2(KON)-P1(KON))/DS
0520      IF (S.LE.0.0001) GO TO 810
0521      PFAC(KON)=(PS(KON)-XNSP(I)*XN(KON)*PCN(KON)/CNS+PSP*PC(KON)/PPS)/
1          UUS
0522  810 KON=KON-1
0523      NITER=NITER+1
0524      IF (NITER.GT.300) GO TO 6000
0525      TFACT1=XV10-P2(J)
0526      TFACT2=XV50-T2(J)
0527      TFACT=XU25-U2(J)
0528      IF (DABS(TFACT)-XFACT) 822,821,821
0529  821 GO TO 2000

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0530      822  CONTINUE
0531      IF(DABS(TFACT1)-XFACT)      824,821,821
0532      9777 CONTINUE
0533      824  CONTINUE
0534      IF(CABS(TFACT2)-XFACT)      820,821,821
0535      820  IF(CONVER.GT.0.0D0)      GO TO 823
0536      CCNVER=1.0D0
0537      GO TO 2000
0538      823  CONTINUE
0539      IF (S.GE.0.0001) GO TO 830
0540      TST=(1.+TW/TTS0)/2.0
0541      VIS4=(TTS0+CONP)*TST**1.5/(TTS0*TST+CONP)
0542      VIS3=VISCO*VIS4
0543      REY=1.0/(EPS*EPS*VISCO)
0544      XKXK=(GAM-1.)/(GAM+1.)*(TTS0/TW+1.)*BO/(2.*EPS*EPS*VIS3)
0545      IF(AFULL.GT.0.0D0)      GO TO 202
0546      GO TO 222
0547      202  CONTINUE
0548      CNT=BLNK
0549      CWS=BLNK
0550      CSS=BLNK
0551      IF(THIN.EQ.-1.) CNT=BNO
0552      IF(SWFAC.EQ.-1.) CWS=BNO
0553      IF(SSFAC.EQ.-1.) CSS=BNO
0554      WRITE (6,922) CNT, CWS, CSS, IE, IEND, DS
0555      922  FORMAT(1H0,3XA2,17H THIN SHOCK LAYER,3X,A2,10H WALL SLIP,3X,A2,
111H SHOCK SLIP,5X,15HNO STEPS IN N =,I4,
216H NO STEPS IN S =,I4,5H DS =,F5.3)
0556      WRITE (6,924) RMAC, BO, EPS, REYIN, REY
0557      924  FORMAT(1H0,5X4HMINF,7X5HTW/TO,7X3HEPS,7XBHREY(INF),6X6HREY(S)/
13F12.4,2E13.3)
0558      222  CONTINUE
0559      830  REFAC=RPS*VVM*CNS/(EPS*EPS*VISCO)
0560      CFCH=2.*UUS*RRS*VVM*VISC(1)*(UCN(1)-CK*CNS*UC(1))/REFAC
0561      HEAT= TTS*RRS*VVM*(CONO*CCN(1)*TCN(1)/VISCO+UUS*UUS*VISC(1)*UC(1)
1 *UCN(1)/TTS)/REFAC
0562      STAN=HEAT/(0.5+1.0/((GAM-1.0)*RMAC*RMAC)-TW)
0563      XNSP(1)=(XNS-XNS1)/DS
0564      DO 840 N=1,IE
0565      XM(N)=DSQRT((UUS*UUS*UC(N)*UC(N)+VVM*VVM*VC(N)*VC(N))/
1 ((GAM-1.)*TTS*TC(N)))
0566      PO2PD1=1.0
0567      IF (XM(N).LE.1.0) GO TO 845
0568      PO2PD1=((GAM+1.0)*XM(N)*XM(N)/(2.+(GAM-1.)*XM(N)*XM(N))**
1 (GAM/(GAM-1.))/(2.*GAM*XM(N)*XM(N)/(GAM+1.)-(GAM-1.)/(GAM
2 +1.))**1/(GAM-1.))
0569      845  PITD(N)=PO2PD1*PC(N)*PPS*(1.+(GAM-1.)*XM(N)*XM(N)/2.)*(GAM/
1 (GAM-1.))/POIP
0570      IF(1.EQ.1) VC(N)=VC(N)*VVS
0571      IF(1.EQ.1) V2(N)=VC(N)
0572      IF(1.LE.5) VCDI(N,I)=VC(N)
0573      IF(1.EQ.1) VCD1(N,I)=VC(N)/VVS
0574      IF(1.EQ.2) VCD1(N,I)=VC(N)/VVS

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0575      U1(N)=U2(N)
0576      V1(N)=V2(N)
0577      T1(N)=T2(N)
0578      R1(N)=R2(N)
0579      P1(N)=P2(N)
0580      T1N(N)=T2N(N)
0581      T1NN(N)=T2NN(N)
0582      U1N(N)=U2N(N)
0583      U1NN(N)=U2NN(N)
0584      C11(N)=C12(N)
0585      840 CD1(N)=CD2(N)
0586      PWALL=PPS*PC(1)
0587      IF (SWFAC) 843,843,844
0588      844 PWALL=PWALL-BSL*EPS**2*DSQRT((GAM-1.0D0)/(GAM*TTS*TC(1)))*VISCO
1          *VISC(1)*TTS*TCN(1)/CNS
0589      843 CONTINUE
0590      IF (S.LE.0.0001) GO TO 841
0591      CDP2=4.*RS*SIF*PWALL
0592      CDF2=2.*RS*CSF*CFCH
0593      CDPD=CDPD+(CDP1+CDP2)*DS/2.
0594      CDFD=CDFD+(CDF1+CDF2)*DS/2.
0595      CDP=CDPD/(RS*RS)
0596      CDF=CDFD/(RS*RS)
0597      841 IF (S.GE.0.0001) GO TO 842
0598      PWALO=PWALL
0599      CDF=0.0
0600      CDP=2.0*PWALL
0601      842 CDTOT=CDF+CDP
0602      CDP1=CDP2
0603      CDF1=CDF2
0604      PWRAT=PWALL/PWALO
0605      XNS1=XNS
0606      UUS1=UUS2
0607      VVS1=VVS2
0608      TTS1=TTS2
0609      PPS1=PPS2
0610      RRS1=RRS2
0611      IL=IEND-1
0612      IF(I.EQ.IEND)      XNS21=RSH1
0613      IF(I.EQ.IEND)      XNS20=RSH
0614      IF(AFULL.GT.0.0D0)      GO      TO      203
0615      GO      TO      223
0616      203 CONTINUE
0617      WRITE (6,926) S,XB,RS,CNS,XNSP(I),XSH,RSH,NITER
0618      WRITE (6,928) UUS, VVS, TTS, RRS, PPS
0619      928 FORMAT(1H0, 8X3HUSH,10X3HVS,10X3HTSH,10X3HRSH,10X3HPSH/3X6F13.6)
0620      WRITE(6,864)      USP,VSP,TSP,RSP,PSP
0621      864 FORMAT(1H0, 8X3HUSP,10X3HVS,10X3HTSP,10X3HRSP,10X3HPSP/3X6F13.6)
0622      WRITE(6,927) CFCH,HEAT,STAN,CDF,CDP,CDTOT,PWALL,PWRAT
0623      927 FORMAT(1H0,5X,2HCF,10X4HHEAT,8X4HSTAN,8X3HCDF,9X3HCDP,9X5HCDTOT,7X
15HPWALL,7X5HPW/P0/8F12.6)
0624      926 FORMAT(1H0,5X,1HS,11X1HX,11X1HR,11X3HNHSH,9X4HNSHP,8X3HXSH,9X3HRSH,
15X,7HNO ITER/7F12.6,16)

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0625      556 FORMAT(3X,3F10.6)
0626      GO TO 223
0627      930 FORMAT(1H0, 8X5HU/USH,8X5HV/VSH,8X5HT/TSH,8X5HR/RSH,4X11HP/PSH(APP
          1R),6X5HP/PSH,8X5HN/NSH,8X4HMACH,9X4HPITO)
0628      932 FORMAT(3X,9F13.6)
0629      223 CONTINUE
0630          IF(I.EQ.NJ1) AM=1.0D0+XNS
0631          AAK3=1.0D0
0632          IF(I.EQ.NJ1) AAK5=2.0D0*AM*DTAN(ALP-PHI)
0633          IF(I.EQ.1) GO TO 999
0634          AA3=-YNSPP(I)+2.0D0*CK*DTAN(ALP3-PHI3)*YNSP(I)+2.0D0*(2.0D0*RSH-
          1 YNSH(I))/DT
0635          AAA1(I)=-2.0D0*CK*DTAN(ALP3-PHI3)
0636          AAA2(I)=-2.0D0/DT
0637          AAA3(I)=AA3
0638          AAA4(I)=0.0D0
0639          IF(I.EQ.NJ1) ALP3A=(3.0D0*AAA3(NJ1)-AAA3(NJ1-1))/2.0D0
0640          IF(I.EQ.NJNC1) ALP3B=(3.0D0*AAA3(NJNC)-AAA3(NJNC+1))/2.0D0
0641      999 CONTINUE
0642      RS=RS2
0643      S=S+DS2
0644      CALL GEOM(S,DS2,RS,CK,CSF,SIF,XB)
0645      PHI3=DARCOS(CSF)
0646      PHI1=PHI3
0647          IF(CONE.GT.0.0D0) PHIS=0.0D0
0648          IF(CONE.GT.0.0D0) PHIS2=0.0D0
0649          IF(CONE.LT.0.0D0) PHIS=-1.0D0
0650          IF(CONE.LT.0.0D0) PHIS2=-1.0D0
0651      ALP3=DATAN ( YNSP(I+1)/AXSP(I+1))
0652      SP3=DSIN(ALP3)
0653      CP3=DCOS(ALP3)
0654      SPB3=SP3*SIF+CP3*CSF
0655      CPB3=CP3*SIF-SP3*CSF
0656      PHIS=(PHI3-PHI1)/DS*2.0D0
0657      5010 FORMAT(6F10.4)
0658      RS2=RS
0659      NITER=0
0660      CONVER=-1.0D0
0661          IF(I.EQ.1) CNS0=CNS
0662      5000 CONTINUE
0663      CN+1 TIME SWEEP STARTS
0664          ENDNSH=(XNS21-XNS20)+XNS21
0665          CALL BOUND1(EF1,FF1)
0666          EF1=0.0D0
0667          FF1=0.0D0
0668          AAK4=ALP3B-ALP3A
0669          IEND1=IEND+1
0670          CALL MANISH(YNSPP,YNSP,YNSH,ENDNSH,DS,AAK3,AAK4,AAK5,AM)
0671          CALL PUSHPA(YNSH,DS,IEND,YNSP)
0672          TIME=TIME+DT
0673          IF(AFULL.GT.0.0D0.AND.THIN.GT.0.0D0) GO TO 6005
0674          IF(THETA.GT.THETA1) GO TO 8010
0675          IF(AFULL.GT.0.0D0) GO TO 6005

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0675      DO      8009      N=1,IEND1
0676      CONV=DABS(YNSH(N)-CNS2(N))
0677      IF(CONV.GT.0.001D0)      GO      TO      8010
0678      8009      CONTINUE
0679      CONV2=DABS(CNS0-XNSH(1))
0680      GO      TO      18
0681      8010      CONTINUE
0682      DO      78      N=1,IEND1
0683      YNSH(N)=WW*CNS2(N)+YNSH(N)*(1.0-WW)
0684      YNSP(N)=WW*CNS2P(N)+YNSP(N)*(1.0-WW)
0685      YNSPP(N)=WW*CNS2PP(N)+YNSPP(N)*(1.0-WW)
0686      78      CONTINUE
0687      CALL      PUSHPA(YNSH,DS,IEND,YNSP)
0688      IF(NTIME1.GT.2.AND.THETA.GT.THETA1)      THETA=THETA -DTHETA
0689      IF(NTIME1.GT.2.AND.THETA.GT.THETA1)      NTIME1=0
0690      GO      TO      77
0691      18      CONTINUE
0692      IF(AFULL.LT.0.0D0)      NTIME=0
0693      AFULL=1.0D0
0694      GO      TO      77
0695      6005      CONTINUE
0696      IF(THIN.EQ.-1.0D0)      GO      TO      19
0697      IEND=IEND-1
0698      THIN=THINI
0699      NTIME=0
0700      AFULL=-1.0D0
0701      GO      TO      77
0702      5050      CONTINUE
0703      19      CONTINUE
0704      DO 1200      N=1,IEND1
0705      WRITE(7,1201)      YNSH(N),YNSP(N),YNSPP(N),AXSP(N),XNSH(N),XNSP(N)
0706      1200      CONTINUE
0707      1201      FORMAT(6F12.8)
0708      6000      CONTINUE
0709      STOP
0710      END

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PEQSO

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0001      SUBROUTINE PEQSO(W1NN,W1N,W1,W2NN,W2N,W2,E1,F1,CRNI,CS1,CS2,SSFAC
1,END,IBACK)
0002      IMPLICIT REAL*8 (A-H, O-Z)
0003      COMMON /PEQS/ DS, DN(201), IM, IE, A1(201), A2(201), A3(201), A4(201),
1          XN(202)
0004      DIMENSION W1NN(201), W1N(201), W1(201)
0005      DIMENSION E(201), F(201), W2NN(201), W2N(201), W2(201)
0006      E(1)=E1
0007      F(1)=F1
0008      DO 10 N=2, IM
0009      A=(2.0D0-A1(N)*DN(N))/(DN(N-1)*(DN(N)+DN(N-1)))*CRNI
0010      B=(-2.0D0+A1(N)*(DN(N)-DN(N-1)))/(DN(N)*DN(N-1)+A2(N))*CRNI
1      +A4(N)/DS
0011      C=(2.0D0+A1(N)*DN(N-1))/(DN(N)*(DN(N)+DN(N-1)))*CRNI
0012      D=-(W1NN(N)+A1(N)*W1N(N)+A2(N)*W1(N))*(1.0D0-CRNI)
1      -A3(N)+A4(N)*W1(N)/DS
0013      E(N)=-C/(B+A*E(N-1))
0014      10 F(N)=(D-A*F(N-1))/(B+A*E(N-1))
0015      IF (SSFAC) 11, 11, 12
0016      12 SK1=CS1+(DN(IM-1)+2.0D0*DN(IM))/(DN(IM)*(DN(IM)+DN(IM-1)))
1      -(DN(IM-1)+DN(IM))*E(IM)/(DN(IM-1)*DN(IM))
2      -DN(IM)*(B*E(IM)+C)/(A*DN(IM-1)*(DN(IM)+DN(IM-1)))
0017      SK2=-CS2+(DN(IM-1)+DN(IM))*F(IM)/(DN(IM-1)*DN(IM))
1      +DN(IM)*(B*F(IM)-D)/(A*DN(IM-1)*(DN(IM-1)+DN(IM)))
0018      W2(IE)=SK2/SK1
0019      GO TO 13
0020      11 W2(IE)=END
0021      13 KON=IM
0022      DO 20 N=2, IE
0023      W2(KON)=E(KON)*W2(KON+1)+F(KON)
0024      20 KON=KON-1
0025      W2(1)=E1*W2(2)+F1
0026      GO TO (21,5), IBACK
0027      21 CONTINUE
0028      DO 30 N=2, IM
0029      W2NN(N)=2.0D0*(W2(N+1)/DN(N)+W2(N-1)/DN(N-1))/(DN(N)+DN(N-1))
1      -2.0D0*W2(N)/(DN(N)*DN(N-1))
0030      30 W2N(N)=(DN(N-1)*W2(N+1)/DN(N)-DN(N)*W2(N-1)/DN(N-1))/(DN(N)+
1      DN(N-1))+((DN(N)-DN(N-1))*W2(N)/(DN(N)*DN(N-1))
0031      W2N(1)=-W2(1)*(DN(2)+2.0D0*DN(1))/(DN(1)*(DN(2)+DN(1)))
1      +W2(2)*(DN(2)+DN(1))/(DN(2)*DN(1))
2      -W2(3)*DN(1)/(DN(2)*(DN(1)+DN(2)))
0032      W2N(IE)=W2(IE)*(DN(IM-1)+2.0D0*DN(IM))/(DN(IM)*(DN(IM)+DN(IM-1)))
1      -W2(IE-1)*(DN(IM-1)+DN(IM))/(DN(IM)*DN(IM-1))
2      +W2(IE-2)*DN(IM)/(DN(IM-1)*(DN(IM)+DN(IM-1)))
0033      W2NN(1)=-W2N(1)*(DN(2)+2.0D0*DN(1))/(DN(1)*(DN(2)+DN(1)))
1      +W2N(2)*(DN(2)+DN(1))/(DN(2)*DN(1))
2      -W2N(3)*DN(1)/(DN(2)*(DN(1)+DN(2)))
0034      W2NN(IE)=W2N(IE)*(DN(IM-1)+2.0D0*DN(IM))/(DN(IM)*(DN(IM)+DN(IM-1)))
2      +W2N(IE-2)*DN(IM)/(DN(IM-1)*(DN(IM)+DN(IM-1)))
1      -W2N(IE-1)*(DN(IM-1)+DN(IM))/(DN(IM)*DN(IM-1))
0035      GO TO 100
0036      5 CONTINUE

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0037      W2N(1)=0.0000
0038      DO      101      N=2, IE
0039      W2N(N)=(W2(N)-W2(N-1))/DN(N-1)
0040      W2NN(N)=(W2N(N)-W2N(N-1))/DN(N-1)
0041      101      CONTINUE
0042      W2NN(1)=(W2N(2)-W2N(1))/DN(1)
0043      100      CONTINUE
0044      RETURN
0045      END
```

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BOUND

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0001      SUBROUTINE BOUND(WINN,WIN,W1,CB1,CB2,E1,F1,CRNI)
0002      IMPLICIT REAL*8 (A-H, O-Z)
0003      COMMON /PEQS/ DS,DN(201),IM,IE,A1(201),A2(201),A3(201),A4(201),
1          XN(202)
0004      DIMENSION WINN(201),WIN(201),W1(201)
0005      A=(2.0D0-A1(2)*DN(2))/(DN(1)*(DN(2)+DN(1)))*CRNI
0006      B=(((-2.0D0+A1(2)*(DN(2)-DN(1)))/(DN(2)*DN(1))+A2(2))*CRNI+A4(2)/DS
0007      C=(2.0D0+A1(2)*DN(1))/(DN(2)*(DN(2)+DN(1)))*CRNI
0008      D=-(W1NN(2)+A1(2)*WIN(2)+A2(2)*W1(2))*(1.0D0-CRNI)
1      -A3(2)+A4(2)*W1(2)/DS
0009      XK1=CB1-(DN(2)+2.0D0C*DN(1))/(DN(1)*(DN(2)+DN(1)))+A*DN(1)/(C*DN(2)
1      *(DN(2)+DN(1)))
0010      XK2=(DN(2)+DN(1))/(DN(2)*DN(1))+B*DN(1)/(C*DN(2)*(DN(2)+DN(1)))
0011      XK3=CB2-D*DN(1)/(C*DN(2)*(DN(2)+DN(1)))
0012      E1=-XK2/XK1
0013      F1=-XK3/XK1
0014      RETURN
0015      END

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GEOM

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0001      SUBROUTINE GEOM(S,DS,RS,CK,CSF,SIF,XB)
          C      SPHERE-CONE
          IMPLICIT REAL*8 (A-H, O-Z)
          COMMON/BASU/      XS3 ,      CONE
          COMMON/PUSHY/      DERIV1,THMAX
          F=12.5000
          XS=S+DS
          XS3=XS
          SMAX=3.141592653589793200/2.000-THMAX
          IF(XS.GT.SMAX)      GO      TO      100
          RS=DSIN(XS)
          CK=1.000-1.000/(1.000+DEXP(-F*(XS-SMAX)))
          CK=1.00
          CONE=-1.000
          CSF=RS
          SIF=DCOS(XS)
          XB=1.000-DCOS(XS)
          GO      TO      200
          100      CONTINUE
          XS1=XS-SMAX
          CSF=DCOS(THMAX)
          SIF=DSIN(THMAX)
          CK=1.000-1.000/(1.000+DEXP(-F*(XS-SMAX)))
          CK=0.000
          CONE=1.000
          RS=DSIN(SMAX)+XS1*DSIN(THMAX)
          XB=1.000-DCOS(SMAX)+XS1*DCOS(THMAX)
          300      FORMAT(4F10.5)
          200      CONTINUE
          RETURN
          END
0030

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SHVALS

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0001      SUBROUTINE SHVALS (SP, CP, SPB, CPB, TTSH, VRSH, URSH, PPSH, ID)
0002      IMPLICIT REAL*8 (A-H, O-Z)
0003      COMMON /INSH/ CONO , GAM , S , UPSH , XNS ,
1          EPS , RMAC , TPSH , VISCO
0004      COMMON/OUTSH/ PPS , RRS , TTS , UUS1 , VVS ,
1          PPS1 , RRS1 , TTS1 , UUS2 , VVS1 ,
2          PSP , RRS2 , TSP , USP , VVS2 ,
3          PPS2 , RSP , TTS2 , UUS , VSP
0005      COMMON /PEQS/ DS,DN(201),IM,IE,A1(201),A2(201),A3(201),A4(201),
1          XN(202)
0006      GAMP = GAM + 1.0D0
0007      GAMM = GAM - 1.0D0
0008      RMACQ = RMAC * RMAC
0009      FOGQ = 4.0D0/(GAMP*GAMP)
0010      SPQ = SP * SP
0011      EPSQ = EPS * EPS
0012      DEN = RMACQ * RMACQ * SPQ
0013      URSH = SP * CP / (SP + EPSQ * VISCO * UPSH/XNS)
0014      TTSH = ((URSH-CP)**2 + FOGQ*GAM*SPQ + (2.0D0/GAMM-FOGQ*GAMM)/RMACQ
1          - FOGQ/DEN)*0.5D0*SP/(SP+EPSQ*CONO*TPSH/XNS)
0015      PPSH = (2.0D0*SPQ - GAMM/(GAM*RMACQ)) / GAMP
0016      RRSH = GAM * PPSH / (GAMM * TTSH)
0017      VRSH = -SP / RRSH
0018      GO TO (20,5) , ID
0019      5 CONTINUE
0020      TTS2 = TTSH
0021      PPS2 = PPSH
0022      RRS2 = RRSH
0023      UUS2 = URSH * SPB + VRSH * CPB
0024      VVS2 = -URSH * CPB + VRSH * SPB
0025      IF (S .GE. .0001) GO TO 10
0026      UUS1 = -UUS2
0027      VVS1 = VVS2
0028      TTS1 = TTS2
0029      PPS1 = PPS2
0030      RRS1 = RRS2
0031      10 CONTINUE
0032      UUS = (UUS1 + UUS2) / 2.0D0
0033      VVS = (VVS1 + VVS2) / 2.0D0
0034      TTS = (TTS1 + TTS2) / 2.0D0
0035      PPS = (PPS1 + PPS2) / 2.0D0
0036      RRS = (RRS1 + RRS2) / 2.0D0
0037      USP = (UUS2 - UUS1) / DS
0038      VSP = (VVS2 - VVS1) / DS
0039      TSP = (TTS2 - TTS1) / DS
0040      PSP = (PPS2 - PPS1) / DS
0041      RSP = (RRS2 - RRS1) / DS
0042      20 CONTINUE
0043      RETURN
0044      END

```

FORTRAN IV G LEVEL 21

BOUND1

DATE = 76296

05/03/31

```
0001      SUBROUTINE BOUND1(EE1,FF1)
0002      IMPLICIT REAL*8 (A-H, O-Z)
0003      COMMON /PEQS/ DS, DN(201), IM, IE, A1(201), A2(201), A3(201), A4(201),
1          XN(202)
0004      A=1.0D0
0005      B=-2.0D0+DS*DS*A2(2)
0006      C=1.0D0
0007      D=-DS*DS*A3(2)
0008      EE1=(B+4.0D0*C)/(3.0D0*C-A)
0009      FF1=-D/(3.0D0*C-A)
0010      RETURN
0011      END
```

FORTRAN IV G LEVEL 21

PUSHPA

DATE = 76299

23/14/20

```

0001      SUBROUTINE  PUSHPA(YNSH,DS,IEND,YNSP)
0002      IMPLICIT REAL*8 (A-H, O-Z)
0003      COMMON/KINNI/  XNSH(110),XNSP(110),XNSPP(110)
0004      COMMON/PUSHY/  DERIV1,THMAX
0005      COMMON/CON/    NJNC,NJ1,RUMP
0006      DIMENSION     YNSH(110),YNSP(110)
0007      DIMENSION     ACKP(110) ,APH(110)
0008      COMMON/BASU/   XS3 ,      CONE
0009      COMMON/MANIS/  AXSH(110),AXSP(110),AXSPP(110)
0010      XS=0.000
0011      IEND1=IEND+1
0012      DO 500      I=2,IEND1
0013      CALL      GEOM (XS,DS,RS,CK,CSF,SIF,XB)
0014      XNSH(I)=(YNSH(I)-RS)/CSF
0015      AXSH(I)=XB-XNSH(I)*SIF
0016      ACKP(I)=(1.000+CK*XNSH(I))
0017      APH(I)=DARCOS(CSF)
0018      XS=XS3
0019      500      CONTINUE
0020      XNSH(1)=(4.000*XNSH(2)-XNSH(3))/3.000
0021      AXSH(1)=-XNSH(1)
0022      DC 12      N=2,IEND
0023      IF(RUMP.LT.0.000)      GO TO      510
0024      IF(N.EQ.NJ1)      GO TO      501
0025      IF(N.EQ.NJNC)      GO TO      504
0026      510      CONTINUE
0027      AXSP(N)=(AXSH(N+1)-AXSH(N-1))/(2.000*DS)
0028      GO TO      502
0029      501      CONTINUE
0030      AXSP(NJ1)=(3.000*AXSH(NJ1)-4.000*AXSH(NJ1-1)+AXSH(NJ1-2))/
0031      1(2.000*DS)
0032      GO TO      503
0033      504      CONTINUE
0034      AXSP(NJNC)=(4.000*AXSH(NJNC+1)-AXSH(NJNC+2)-3.000*AXSH(NJNC))/
0035      1(2.000*DS)
0036      503      CONTINUE
0037      502      CONTINUE
0038      12      CONTINUE
0039      XNSP(1)=0.000
0040      AXSP(1)=0.000
0041      AXSP(IEND1)=(3.000*AXSH(IEND1)-4.000*AXSH(IEND1-1)+AXSH(IEND1-2))
0042      1/(2.000*DS)
0043      DO 860      I=1,IEND1
0044      IF(I.EQ.1)      GO TO      81
0045      TALP=YNSP(I)/AXSP(I)
0046      XNSP(I)=ACKP(I)*((TALP-DTAN(APH(I)))/(1.0+TALP*DTAN(APH(I))))
0047      GO TO      511
0048      512      CONTINUE
0049      IF(I.LT.IEND1)      XNSP(I)=(XNSH(I+1)-XNSH(I-1))/(DS*2.000)
0050      IF(I.EQ.IEND1)      XNSP(I)=(3.000*XNSH(IEND1)-4.000*XNSH(IEND1-1)
0051      1+XNSH(IEND1-2))/(2.000*DS)
0052      511      CONTINUE
0053      81      CONTINUE

```


FORTRAN IV G LEVEL 21

PUSHPA

DATE = 76299

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```
0050      860    CONTINUE
0051      XNSPP(1)=(2.000*XNSH(1)-5.0000*XNSH(2)+4.000*XNSH(3)-XNSH(4))
          1/(DS*CS)
0052      RETURN
0053      END
```

FORTRAN IV G LEVEL 21

DERIV

DATE = 76296

05/03/31

```

0001      SUBROUTINE      DERIV(DS,IEND,IEND1,AXNSH,AXNSP,AXNSPP)
0002      IMPLICIT REAL*8 (A-H, O-Z)
0003      DIMENSION        X1SP(110),A1SP(110),X1SH(110)
0004      DIMENSION        YNSH(110),YNSP(110),YNSPP(110)
0005      DIMENSION        ACKP(110) ,APH(110)
0006      COMMON/CON/      NJNC,NJ1,RUMP
0007      COMMON/BASU/     XS3 ,      CONE
0008      COMMON/KINNI/    XNSH(110),XNSP(110),XNSPP(110)
0009      COMMON/MANIS/    AXSH(110),AXSP(110),AXSPP(110)
0010      COMMON/PUSHY/    DERIV1,THMAX
0011      DIMENSION        AXNSH(119),AXNSP(119),AXNSPP(119)
0012      DERIV1=1.0D0
0013      AHALF=1.0D0
0014      AHALF=-1.0D0
0015      IEND1=IEND+1
0016      READ(5,80)      (XNSH(I),I=1,8)
0017      READ(5,80)      (XNSH(I),I=9,16)
0018      READ(5,81)      (XNSH(I),I=17,22)
0019      READ(5,80)      (XNSH(I),I=23,30)
0020      READ(5,80)      (XNSH(I),I=31,38)
0021      READ(5,81)      (XNSH(I),I=39,44)
0022      READ(5,80)      (XNSH(I),I=45,52)
0023      READ(5,80)      (XNSH(I),I=53,60)
0024      READ(5,82)      XNSH(61)
0025      82  FORMAT(F10.6)
0026      80  FORMAT(8F10.6)
0027      81  FORMAT(6F10.6)
0028      XS=0.0D0
0029      DO      500      I=2,IEND1
0030      CALL      GEOM (XS,DS,RS,CK,CSF,SIF,XB)
0031      YNSH(I)=RS+XNSH(I)*CSF
0032      AXSH(I)=XB-XNSH(I)*SIF
0033      ACKP(I)=(1.0D0+CK*XNSH(I))
0034      APH(I)=DARCOS(CSF)
0035      IF(I.EQ.2)      YS1=RS
0036      XS=XS3
0037      500      CONTINUE
0038      YNSH(1)=0.0D0
0039      AXSH(1)=-XNSH(1)
0040      DO      12      N=2,IEND
0041      IF(RUMP.LT.0.0D0)      GO      TO      510
0042      IF(N.EQ.NJ1)      GO      TO      501
0043      IF(N.EQ.NJNC)      GO      TO      504
0044      510      CONTINUE
0045      YNSP(N)=(YNSH(N+1)-YNSH(N-1))/(2.0D0*DS)
0046      AXSP(N)=(AXSH(N+1)-AXSH(N-1))/(2.0D0*DS)
0047      GO      TO      502
0048      501      CONTINUE
0049      AXSP(NJ1)=(3.0D0*AXSH(NJ1)-4.0D0*AXSH(NJ1-1)+AXSH(NJ1-2))/
1(2.0D0*DS)
0050      YNSP(NJ1)=(3.0D0*YNSH(NJ1)-4.0D0*YNSH(NJ1-1)+YNSH(NJ1-2))/
1(2.0D0*DS)
0051      GO      TO      503

```


FORTRAN IV G LEVEL 21

DERIV

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0052      504      CONTINUE
0053          AXSP(NJNC)=(4.0D0*AXSH(NJNC+1)-AXSH(NJNC+2)-3.0D0*AXSH(NJNC))/
          1(2.0D0*DS)
0054          YNSP(NJNC)=(4.0D0*YNSH(NJNC+1)-YNSH(NJNC+2)-3.0D0*YNSH(NJNC))/
          1(2.0D0*DS)
0055      503      CONTINUE
0056      502      CONTINUE
0057      12       CONTINUE
0058          YNSP(1)=(4.0D0*YNSH(2)-YNSH(3)-3.0D0*YNSH(1))/(2.0D0*DS)
0059          YNSP(1)=XNSH(1)+YS1/DS
0060          YNSP(1)=XNSH(1)+1.0D0
0061          XNSP(1)=0.0D0
0062          AXSP(1)=0.0D0
0063          AXSP(IEND1)=(3.0D0*AXSH(IEND1)-4.0D0*AXSH(IEND1-1)+AXSH(IEND1-2))/
          1(2.0D0*DS)
0064          YNSP(IEND1)=(3.0D0*YNSH(IEND1)-4.0D0*YNSH(IEND1-1)+YNSH(IEND1-2))/
          1(2.0D0*DS)
0065          DO 700 I=1,IEND1
0066              IF(I.EQ.1) GO TO 600
0067              TALP=YNSP(I)/AXSP(I)
C          IF(RUMP.LT.0.0D0) GO TO 512
0068              XNSP(I)=ACKP(I)*((TALP-DTAN(APH(I)))/(1.0+TALP*DTAN(APH(I))))
0069              GO TO 511
0070      512      CONTINUE
0071              IF(I.LT.IEND1) XNSP(I)=(XNSH(I+1)-XNSH(I))/(2.0D0*DS)
0072              IF(I.EQ.IEND1) XNSP(I)=(3.0D0*XNSH(IEND1)-4.0D0*XNSH(IEND1-1)
          1+XNSH(IEND1-2))/(2.0D0*DS)
0073      511      CONTINUE
0074      600      CONTINUE
0075      700      CONTINUE
0076          DO 28 N=2,IEND
C          IF(RUMP.LT.0.0D0) GO TO 530
0077              IF(N.EQ.NJ1) GO TO 531
0078              IF(N.EQ.NJNC) GO TO 532
0079      530      CONTINUE
0080              YNSPP(N)=(YNSH(N+1)+YNSH(N-1)-2.0D0*YNSH(N))/(DS*DS)
0081              GO TO 533
0082      531      CONTINUE
0083              YNSPP(N)=(2.0D0*YNSH(N)+4.0D0*YNSH(N-2)-5.0D0*YNSH(N-1)-YNSH(N-3))/
          1/(DS*DS)
0084              GO TO 533
0085      532      CONTINUE
0086              YNSPP(N)=(2.0D0*YNSH(N)-5.0D0*YNSH(N+1)+4.0D0*YNSH(N+2)-YNSH(N+3))/
          1/(DS*DS)
0087      533      CONTINUE
0088      28       CONTINUE
0089          YNSPP(1)=(4.0D0*YNSP(2)-YNSP(3)-3.0D0*YNSP(1))/(2.0D0*DS)
0090          YNSPP(1)=0.0D0
0091          YNSPP(IEND1)=(3.0D0*YNSP(IEND1)-4.0D0*YNSP(IEND1-1)+YNSP(IEND1-2))/
          1/(2.0D0*DS)
0092          YNSPP(IEND1)=(2.0D0*YNSH(IEND1)-5.0D0*YNSH(IEND1-1)+4.0D0*YNSH(
          1IEND1-2)-YNSH(IEND1-3))/(DS*DS)
C          *****

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FORTRAN IV G LEVEL 21

DERIV

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0093      IF(AHALF.LT.0.000)      GO      TO      6006
0094      APARA=-1.000
0095      APARA=1.000
0096      6019      CONTINUE
0097      DO      6009      I=1,IEND
0098      AXNSH(I+I)=(YNSH(I)+YNSH(I+1))/2.000
0099      AXNSP(I+I)=(YNSP(I)+YNSP(I+1))/2.000
0100      AXNSPP(I+I)=(YNSPP(I)+YNSPP(I+1))/2.000
0101      A1SP(I+I)=(AXSP(I)+AXSP(I+1))/2.000
0102      X1SH(I+I)=(XNSH(I)+XNSH(I+1))/2.000
0103      X1SP(I+I)=(XNSP(I)+XNSP(I+1))/2.000
0104      6009      CONTINUE
0105      NJJ1=NJ1+NJ1
0106      I2END=2*IEND+1
0107      I I=1
0108      DO      6007      I=1,I2END,2
0109      AXNSH(I)=YNSH(I)
0110      AXNSP(I)=YNSP(I)
0111      AXNSPP(I)=YNSPP(I)
0112      X1SH(I)=XNSH(I)
0113      X1SP(I)=XNSP(I)
0114      A1SP(I)=AXSP(I)
0115      I I=I+1
0116      6007      CONTINUE
0117      NJ1=NJJ1
0118      NJNC=NJJ1+1
0119      IEND1=I2END
0120      IEND=IEND1-1
0121      DO      6011      N=1,IEND1
0122      XNSH(N)=X1SH(N)
0123      AXSP(N)=A1SP(N)
0124      XNSP(N)=X1SP(N)
0125      6011      CONTINUE
0126      AXNSH(NJJ1)=(2.000*AXNSH(NJJ1-1)-AXNSH(NJJ1-2))
0127      AXNSP(NJJ1)=(2.000*AXNSP(NJJ1-1)-AXNSP(NJJ1-2))
0128      AXNSPP(NJJ1)=(2.000*AXNSPP(NJJ1-1)-AXNSPP(NJJ1-2))
0129      XNSH(NJJ1)=(2.000*XNSH(NJJ1-1)-XNSH(NJJ1-2))
0130      XNSP(NJJ1)=(2.000*XNSP(NJJ1-1)-XNSP(NJJ1-2))
0131      AXSP(NJJ1)=(2.000*AXSP(NJJ1-1)-AXSP(NJJ1-2))
0132      IF(APARA.GT.0.000)      GO      TO      6020
0133      DO      6021      I=1,IEND1
0134      YNSH(I)=AXNSH(I)
0135      YNSP(I)=AXNSP(I)
0136      YNSPP(I)=AXNSPP(I)
0137      6021      CONTINUE
0138      GO      TO      6019
0139      6020      CONTINUE.
0140      6006      CONTINUE
0141      IF(AHALF.GT.0.000)      GO      TO      6010
C *****
0142      DO      1200      N=1,IEND1
0143      READ(5,1201)      YNSH(N),YNSP(N),YNSPP(N),AXSP(N),XNSH(N),XNSP(N)
0144      1200      CONTINUE

```

FORTRAN IV G LEVEL 21

DERIV

DATE = 76296

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```
0145      XNSPP(1)=(2.0D0*XNSH(1)-5.00D0*XNSH(2)+4.0D0*XNSH(3)-XNSH(4))
          1/(DS*DS)
0146      DO 6008 I=1,IEND1
0147          AXNSH(I)=YNSH(I)
0148          AXNSP(I)=YNSP(I)
0149          AXNSPP(I)=YNSPP(I)
0150      6008      CONTINUE
0151      6010      CONTINUE
0152      1201      FORMAT(6F12.8)
0153          ANSH=YNSH(IEND1)
0154          RETURN
0155          END
```


FORTRAN IV G LEVEL 21

MANISH

DATE = 76296

05/03/31

```

0001      SUBROUTINE MANISH(ARXX,ARX,AR,RL,DX,AAK3,AAK4,AAK5,AM)
0002      IMPLICIT REAL*8 (A-H, O-Z)
0003      COMMON/MANU/EE1,FF1,IEND,IEND1,AAA1(110),AAA2(110),AAA3(110)
0004      1,AAA4(110)
0005      COMMON/CON/ NJNC,NJ1,RUMP
0006      DIMENSION ARXX(110),ARX(110),AR(110)
0007      DIMENSION E(120),F(120)
0008      E(1)=EE1
0009      F(1)=FF1
0010      XI=0.50D0
0011      RO=0.0D0
0012      IEND1=IEND+1
0013      IM=IEND1-1
0014      IF(RUMP.GT.0.0D0) GO TO 501
0015      AM=1.0D0
0016      AAK3=1.0D0
0017      AAK1=1.0D0
0018      AAK4=0.0D0
0019      AAK5=0.0D0
0020      501 CONTINUE
0021      DO 10 N=2,IM
0022      ALP1B=AAA1(N)
0023      ALP1A=AAA1(N)
0024      ALP2=AAA2(N)
0025      ALP3B=AAA3(N)
0026      ALP3A=AAA3(N)
0027      AK1=1.0D0
0028      AK3=1.0D0
0029      AK4=0.0D0
0030      AK5=0.0D0
0031      IF(N.LT.NJ1) GO TO 11
0032      IF(N.GT.NJNC) GO TO 11
0033      AK1=AM
0034      AK3=AAK3
0035      AK4=AAK4
0036      AK5=AAK5
0037      IF(N.EQ.NJ1) GO TO 12
0038      AP1=XI*(AK1-DX*X1*AK5/2.0D0)+(1.0D0-XI)
0039      AP2=XI*(1.0D0-XI)*(AK1-DX*X1*AK5/2.0D0)+X1*X1*AK3/2.0D0+
0040      1(1.0D0-XI)**2.0D0/2.0D0
0041      AP3=AP1+2.0D0*AP2
0042      A=2.0D0/(DX*DX*AP3)-ALP1B/(DX*AP3)
0043      B=-2.0D0*(1.0D0+AP1)/(DX*DX*AP3)+ALP1B*(1.0D0-2.0D0*AP2)/(DX*AP3)
0044      1+ALP2
0045      C=AP1*2.0D0/(DX*DX*AP3)+2.0D0*ALP1B*AP2/(DX*AP3)
0046      D=-ALP3B+X1*X1*AK4/AP3-DX*ALP1B*X1*X1*AK4/(2.0D0*AP3)
0047      GO TO 13
0048      12 CONTINUE
0049      AS=(1.0D0-XI)*(1.0D0-DX*(1.0D0-XI)*AK5/(2.0D0*AK3))/AK1
0050      AP1=XI+AS
0051      AP2=XI*X1/2.0D0+X1*AS+(1.0D0-XI)**2.0D0/(2.0D0*AK3)
0052      AP3=AP1+2.0D0*AP2
0053      A=AP1*2.0D0/(DX*DX*AP3)-2.0D0*ALP1A*AP2/(DX*AP3)

```

FORTRAN IV G LEVEL 21

MANISH

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0051      B=-2.0D0*(1.0D0+AP1)/(DX*DX*AP3)-ALP1A*(1.0D0-2.0D0*AP2)/(DX*AP3)
          1+ALP2
0052      C=2.0D0/(DX*DX*AP3)+ALP1A/(DX*AP3)
0053      A14=AK4/AK3
0054      D=-ALP3A-ALP1A*DX*((1.0D0-XI)**2.0D0)*A14/(2.0D0*AP3)-(1.0D0-XI)
          1**2.0D0*A14/AP3
0055      GO TO 14
0056      11 CONTINUE
0057      A=1.0D0/(DX*DX)-AAA1(N)/(2.0D0*DX)
0058      B=-2.0D0/(DX*DX)+AAA2(N)
0059      C=1.0D0/(DX*DX)+AAA1(N)/(2.0D0*DX)
0060      D=-AAA3(N)
0061      14 CONTINUE
0062      13 CONTINUE
0063      E(N)=-C/(B+A*E(N-1))
0064      F(N)=(D-A*F(N-1))/(B+A*E(N-1))
0065      10 CONTINUE
0066      KON=IM
0067      AR(IEND1)=RL
0068      AR(1)=R0
0069      DO 20 N=2,IEND1
0070      AR(KON)=E(KON)*AR(KON+1)+F(KON)
0071      KON=KON-1
20      C CALCULATION OF DERIVATIVES
0072      DO 30 N=2,IM
0073      IF(N.EQ.NJ1) GO TO 31
0074      IF(N.EQ.NJNC) GO TO 40
0075      ARX(N)=(AR(N+1)-AR(N-1))/(2.0D0*DX)
0076      ARXX(N)=(AR(N+1)+AR(N-1)-2.0D0*AR(N))/(DX*DX)
0077      GO TO 50
0078      31 CONTINUE
0079      AK1=AM
0080      AK3=AAK3
0081      AK4=AAK4
0082      AK5=AAK5
0083      AS=(1.0D0-XI)*(1.0D0-DX*(1.0D0-XI)*AK5/(2.0D0*AK3))/AK1
0084      AP1=XI+AS
0085      AP2=XI*XI/2.0D0+XI*AS+(1.0D0-XI)**2.0D0/(2.0D0*AK3)
0086      AP3=AP1+2.0D0*AP2
0087      A14=AK4/AK3
0088      ARX(N)=(AR(N+1)-2.0D0*AR(N-1)*AP2-AR(N)*(1.0D0-2.0D0*AP2)+DX*DX
          1*(1.0D0-XI)*(1.0D0-XI)*A14)/(DX*AP3)
0089      ARXX(N)=(AP1*AR(N-1)+AR(N+1)-AR(N)*(AP1+1.0D0)+DX*DX*(1.0D0-XI)*
          1*(1.0D0-XI)*A14/2.0D0)/(DX*DX*AP3/2.0D0)
0090      GO TO 50
0091      40 CONTINUE
0092      AK1=AM
0093      AK3=AAK3
0094      AK4=AAK4
0095      AK5=AAK5
0096      AP1=XI*(AK1-DX*XI*AK5/2.0D0)+(1.0D0-XI)
0097      AP2=XI*(1.0D0-XI)*(AK1-CX*XI*AK5/2.0D0)+XI*XI*AK3/2.0D0+
          1*(1.0D0-XI)**2.0D0/2.0D0

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FORTRAN IV G LEVEL 21

MANISH

DATE = 76299

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0098      AP3=AP1+2.000*AP2
0099      ARX(N)=(2.000*AP2*AR(N+1)+AR(N)*(1.000-2.000*AP2)-AR(N-1)+DX*DX
0100      1*XI*XI*AK4/2.000)/(DX*AP3)
      ARXX(N)=(AR(N-1)+AR(N+1)*AP1-AR(N)*(1.000+AP1)-DX*DX*XI*XI*AK4
0101      1/2.000)/(DX*DX*AP3/2.000)
0101      50      CONTINUE
0102      30      CONTINUE
0103      ARX(1)=(-3.000*AR(1)+4.000*AR(2)-AR(3))/(2.000*DX)
0104      ARX(IEND1)=(3.000*AR(IEND1)-4.000*AR(IEND1-1)+AR(IEND1-2))/
0105      1(2.000*DX)
0105      ARXX(1)=0.000
0106      ARXX(1)=(2.000*AR(1)-5.000*AR(2)+4.000*AR(3)-AR(4))/(DX*DX)
0107      ARXX(IEND1)=(2.000*AR(IEND1)-5.000*AR(IEND1-1)+4.000*AR(IEND1-2)
0108      1-AR(IEND1-3))/(DX*DX)
0108      RETURN
0109      END

```


INPUT DATA							
3.500000	11.000000	7.500000	0.800000	0.000010	1.400000	0.700000	2 12
NO THIN SHOCK LAYER NO WALL SLIP NO SHOCK SLIP NO STEPS IN N = 101 NO STEPS IN S = 20 DS = 0.137							
MINF	TW/TO	EPS	REY(INF)	REY(S)			
13.4100	0.0741	0.1049	0.1520 04	0.1310 03			
S	X	R	NSH	NSHP	XSH	RSH	NO ITER
0.0	0.0	0.0	0.131292	0.0	-0.131292	0.0	22
USH	VSH	TSH	RSH	PSH			
0.0	-0.170417	0.497582	5.837150	0.829846			
USP	VSP	TSP	RSP	PSP			
0.953739	0.0	0.0	-0.000000	0.0			
CF	HEAT	STAN	CDF	CDP	CDTOT	PWALL	PW/PO
0.0	0.086606	0.182013	0.0	1.842190	1.842190	0.921095	1.000000
S	X	R	NSH	NSHP	XSH	RSH	NO ITER
0.137133	0.009388	0.136704	0.129787	-0.021954	-0.119180	0.154446	22
USH	VSH	TSH	RSH	PSH			
0.118740	-0.166786	0.491211	5.835012	0.818925			
USP	VSP	TSP	RSP	PSP			
0.874223	0.050466	-0.096666	-0.032436	-0.165709			
CF	HEAT	STAN	CDF	CDP	CDTOT	PWALL	PW/PO
0.030719	0.087050	0.182547	0.004213	1.788548	1.792761	0.899922	0.977014
S	X	R	NSH	NSHP	XSH	RSH	NO ITER
0.274266	0.037376	0.270840	0.131982	0.053970	-0.089673	0.306586	28
USH	VSH	TSH	RSH	PSH			
0.232639	-0.155419	0.473214	5.828668	0.788073			
USP	VSP	TSP	RSP	PSP			
0.783952	0.116371	-0.165070	-0.059678	-0.282969			
CF	HEAT	STAN	CDF	CDP	CDTOT	PWALL	PW/PO
0.060506	0.083455	0.175392	0.010444	1.735561	1.746005	0.845565	0.918000
S	X	R	NSH	NSHP	XSH	RSH	NO ITER
0.411399	0.083438	0.399892	0.140863	0.075557	-0.045672	0.456222	31
USH	VSH	TSH	RSH	PSH			
0.339555	-0.135769	0.445626	5.817947	0.740780			
USP	VSP	TSP	RSP	PSP			
0.772023	0.169457	-0.239527	-0.097640	-0.410606			
CF	HEAT	STAN	CDF	CDP	CDTOT	PWALL	PW/PO
0.081402	0.075554	0.158787	0.019760	1.653118	1.672878	0.761914	0.827075
S	X	R	NSH	NSHP	XSH	RSH	NO ITER
0.548532	0.146709	0.521435	0.153348	0.106525	0.015859	0.601396	33
USH	VSH	TSH	RSH	PSH			
0.438613	-0.107593	0.410673	5.802299	0.680864			
USP	VSP	TSP	RSP	PSP			
0.691323	0.236448	-0.278643	-0.133720	-0.477657			

CF	HEAT	STAN	CDF	CDP	CDTOT	PWALL	PW/PO
0.097430	0.066936	0.140674	0.031548	1.547861	1.579408	0.654892	0.710993
S	X	R	NSH	NSHP	XSH	RSH	NO ITER
0.685665	0.226002	0.633188	0.171189	0.153678	0.093502	0.741583	33
USH	VSH	TSH	RSH	PSH			
0.526864	-0.070203	0.371583	5.781352	0.613854			
USP	VSP	TSP	RSP	PSP			
0.607613	0.303819	-0.299625	-0.175594	-0.513623			
CF	HEAT	STAN	CDF	CDP	CDTOT	PWALL	PW/PO
0.104987	0.057308	0.120440	0.044853	1.429514	1.474367	0.538326	0.584442
S	X	R	NSH	NSHP	XSH	RSH	NO ITER
0.822798	0.319827	0.733052	0.196135	0.210140	0.186422	0.876829	32
USH	VSH	TSH	RSH	PSH			
0.601265	-0.022700	0.331978	5.755171	0.545963			
USP	VSP	TSP	RSP	PSP			
0.490385	0.383401	-0.286747	-0.210476	-0.491541			
CF	HEAT	STAN	CDF	CDP	CDTOT	PWALL	PW/PO
0.104886	0.047424	0.099668	0.058590	1.309062	1.367651	0.423802	0.460106
S	X	R	NSH	NSHP	XSH	RSH	NO ITER
0.959931	0.426424	0.819152	0.230085	0.285009	0.294452	1.007627	31
USH	VSH	TSH	RSH	PSH			
0.660195	0.035136	0.294536	5.724023	0.481780			
USP	VSP	TSP	RSP	PSP			
0.371903	0.459203	-0.261860	-0.244099	-0.448872			
CF	HEAT	STAN	CDF	CDP	CDTOT	PWALL	PW/PO
0.098341	0.038033	0.079931	0.071925	1.196499	1.268424	0.321772	0.349336
S	X	R	NSH	NSHP	XSH	RSH	NO ITER
1.097064	0.543789	0.889872	0.276469	0.391476	0.417661	1.135894	60
USH	VSH	TSH	RSH	PSH			
0.704822	0.101709	0.259543	5.686685	0.421797			
USP	VSP	TSP	RSP	PSP			
0.263302	0.520587	-0.238522	-0.286226	-0.408860			
CF	HEAT	STAN	CDF	CDP	CDTOT	PWALL	PW/PO
0.088185	0.029434	0.061860	0.084468	1.099116	1.183583	0.233792	0.253820
S	X	R	NSH	NSHP	XSH	RSH	NO ITER
1.234197	0.669721	0.943883	0.343699	0.589031	0.556204	1.268295	55
USH	VSH	TSH	RSH	PSH			
0.738785	0.173704	0.225124	5.638285	0.362799			
USP	VSP	TSP	RSP	PSP			
0.226261	0.533528	-0.259566	-0.414421	-0.445605			
CF	HEAT	STAN	CDF	CDP	CDTOT	PWALL	PW/PO
0.075887	0.021742	0.045694	0.096233	1.021446	1.117679	0.159441	0.173100
S	X	R	NSH	NSHP	XSH	RSH	NO ITER
1.371330	0.801854	0.980172	0.439152	0.803091	0.714838	1.410617	46

USH	VSH	TSH	RSH	PSH		
0.761990	0.249529	0.191662	5.574810	0.305444		
USP	VSP	TSP	RSP	PSP		
0.187022	0.534116	-0.275811	-0.606137	-0.472741		
CF	HEAT	STAN	CDF	CDP	CDTOT	PWALL PW/PO
0.062026	0.015841	0.033292	0.107395	0.967262	1.074657	0.105711 0.114766
S	X	R	NSH	NSHP	XSH	RSH NO ITER
1.508463	0.937454	1.000395	0.529161	0.509632	0.868384	1.525029 36
USH	VSH	TSH	RSH	PSH		
0.783683	0.280815	0.167276	5.515229	0.263647		
USP	VSP	TSP	RSP	PSP		
0.189939	-0.110097	-0.118247	-0.340903	-0.202664		
CF	HEAT	STAN	CDF	CDP	CDTOT	PWALL PW/PO
0.049193	0.012505	0.026280	0.117948	0.937070	1.055018	0.080785 0.087705
S	X	R	NSH	NSHP	XSH	RSH NO ITER
1.645596	1.073414	1.018294	0.596364	0.470475	0.995573	1.609556 34
USH	VSH	TSH	RSH	PSH		
0.807850	0.265884	0.152159	5.467445	0.237738		
USP	VSP	TSP	RSP	PSP		
0.158379	-0.106792	-0.099747	-0.347341	-0.170948		
CF	HEAT	STAN	CDF	CDP	CDTOT	PWALL PW/PO
0.040721	0.010799	0.022696	0.125728	0.909735	1.035462	0.071915 0.078075
S	X	R	NSH	NSHP	XSH	RSH NO ITER
1.782729	1.209373	1.036193	0.658261	0.432264	1.123453	1.688823 43
USH	VSH	TSH	RSH	PSH		
0.827629	0.251828	0.139646	5.420094	0.216294		
USP	VSP	TSP	RSP	PSP		
0.127609	-0.097378	-0.081236	-0.335642	-0.139215		
CF	HEAT	STAN	CDF	CDP	CDTOT	PWALL PW/PO
0.036198	0.009647	0.020275	0.131422	0.883284	1.014706	0.065597 0.071217
S	X	R	NSH	NSHP	XSH	RSH NO ITER
1.919862	1.345333	1.054093	0.715520	0.402819	1.251939	1.763491 42
USH	VSH	TSH	RSH	PSH		
0.843984	0.238788	0.129191	5.373517	0.198377		
USP	VSP	TSP	RSP	PSP		
0.106790	-0.090286	-0.068657	-0.331244	-0.117652		
CF	HEAT	STAN	CDF	CDP	CDTOT	PWALL PW/PO
0.033008	0.008852	0.018604	0.135843	0.857802	0.993646	0.060989 0.066214
S	X	R	NSH	NSHP	XSH	RSH NO ITER
2.056995	1.481293	1.071992	0.769285	0.381310	1.380881	1.834696 42
USH	VSH	TSH	RSH	PSH		
0.857710	0.226726	0.120330	5.327713	0.183194		
USP	VSP	TSP	RSP	PSP		
0.091665	-0.084696	-0.059486	-0.330612	-0.101929		

CF	HEAT	STAN	CDF	CDP	CDTOT	PWALL	PW/PO
0.030649	0.008257	0.017353	0.139349	0.833314	0.972663	0.057375	0.062290
S	X	R	NSH	NSHP	XSH	RSH	NO ITER
2.194128	1.617253	1.089892	0.819539	0.351615	1.510282	1.902419	43
USH	VSH	TSH	RSH	PSH			
0.869244	0.215709	0.112817	5.283300	0.170320			
USP	VSP	TSP	RSP	PSP			
0.077149	-0.077060	-0.050508	-0.319154	-0.086540			
CF	HEAT	STAN	CDF	CDP	CDTOT	PWALL	PW/PO
0.028774	0.007799	0.016391	0.142159	0.809813	0.951972	0.054544	0.059216
S	X	R	NSH	NSHP	XSH	RSH	NO ITER
2.331261	1.753213	1.107791	0.864317	0.301446	1.640397	1.964714	45
USH	VSH	TSH	RSH	PSH			
0.878704	0.206006	0.106604	5.241906	0.159675			
USP	VSP	TSP	RSP	PSP			
0.059373	-0.063303	-0.039178	-0.277096	-0.067122			
CF	HEAT	STAN	CDF	CDP	CDTOT	PWALL	PW/PO
0.027154	0.007448	0.015654	0.144409	0.787287	0.931696	0.052554	0.057056
S	X	R	NSH	NSHP	XSH	RSH	NO ITER
2.468354	1.889172	1.125691	0.904733	0.288002	1.771081	2.022684	42
USH	VSH	TSH	RSH	PSH			
0.886844	0.197100	0.101215	5.201760	0.150442			
USP	VSP	TSP	RSP	PSP			
0.050353	-0.056688	-0.033461	-0.262395	-0.057324			
CF	HEAT	STAN	CDF	CDP	CDTOT	PWALL	PW/PO
0.025854	0.007137	0.014999	0.146203	0.765712	0.911916	0.050898	0.055258
S	X	R	NSH	NSHP	XSH	RSH	NO ITER
2.605527	2.025132	1.143590	0.950673	0.382001	1.901045	2.086130	48
USH	VSH	TSH	RSH	PSH			
0.895073	0.187512	0.095721	5.155987	0.141031			
USP	VSP	TSP	RSP	PSP			
0.067366	-0.080597	-0.045135	-0.395463	-0.077315			
CF	HEAT	STAN	CDF	CDP	CDTOT	PWALL	PW/PO
0.025167	0.006817	0.014328	0.147680	0.745016	0.892696	0.048489	0.052643

SYMBOLS

c^*	viscosity law constant, $c^* = 198.6^\circ R$
C_f	skin friction coefficient, $2\tau^*/(\rho_\infty^* u_\infty^{*2})$
C_p^*	specific heat of constant pressure
H	nondimensional total enthalpy, H^*/u_∞^{*2}
k	thermal conductivity
M_∞	free stream Mach number
n	coordinate measured normal to the body, nondimensionalized by the body nose radius
n_s	shock stand off distance normal to the body surface
p	nondimensional pressure, $p^*/(\rho_\infty^* u_\infty^{*2})$
q	nondimensional heat transfer, $q^*/(\rho_\infty^* u_\infty^{*3})$
r	nondimensional axisymmetric radius
R	defined as $y_B + n_s \cos\phi$
s	nondimensional surface distance coordinate
st	Stanton number, $st = q_w/(H_o - H_w)$
T	nondimensional temperature, $T = T^*/(u_\infty^{*2}/C_p^*)$
T_∞^*	free stream temperature
u	nondimensional velocity component tangent to the body surface, u^*/u_∞^*
u_∞^*	free stream velocity
\tilde{u}	nondimensional component of velocity aft and tangent to the shock interface
v	nondimensional velocity component normal to the body surface, v^*/u_∞^*
\tilde{v}	nondimensional component of velocity aft and normal to shock interface

x_B	axial distance for body surface measured from stagnation point
x_s	defined as $x_B - n_s \sin \phi$
y_B	normal distance for body surface measured from axis
α	shock angle, see Figure 1
β	angle defined in Figure 1
γ	ratio of specific heats
ϵ	perturbation parameter, $\epsilon = [\mu^* (u_\infty^{*2}/C_p^*) / \rho_\infty^* u_\infty^* a^*]^{1/2}$
κ	nondimensional surface curvature
μ	nondimensional coefficient of viscosity, $\mu = \mu^* / \mu^* (u_\infty^{*2}/C_p^*)$
ρ	nondimensional density, $\rho = \rho^* / \rho_\infty^*$
ρ_∞^*	free stream density
τ	nondimensional shear stress, $\tau^* / (\rho_\infty^* u_\infty^{*2})$
ϕ	body angle defined in Figure 1
σ	Prandtl number, $\sigma = \mu C_p / K$

Subscripts

l	wall value
0	stagnation conditions
s	used for longitudinal derivatives
sh	conditions immediately behind the shock wave
∞	free stream conditions

Superscripts

- physical quantities normalized by their shock values
- * dimensional quantities, also used for first sweep of ADI numerical scheme
- J 0 for plane flow and 1 for axisymmetric flow
- n+1 represents second sweep of ADI numerical scheme